Theories with NIP. List 8.

We work in a monster model $\mathfrak{C}$ of a complete theory T. Let A be a small subset of $\mathfrak{C}$.

Problem 1. Prove the six properties of forking and dividing in arbitrary theories listed in the proposition on page 41 in the notes.

Problem 2. i) Observe that if for any type $\pi(x)$, $\pi(x)$ forks over $A \iff \pi(x)$ divides over A, then A is an extension base.
ii) Let $T = \mathrm{Th}(S^1, R)$, where R is the circular order. Prove that T has NIP and the formula $x = x$ forks over $\emptyset$ (although it clearly does not divide over $\emptyset$).

Problem 3. Let $p, q \in S(\mathfrak{C})$ be invariant types (over some small set of parameters) which do not fork over A. Prove that $p(x) \otimes q(y)$ does not fork over A.

Problem 4. Let $p \in S_n(A)$, where $n < \omega$. Prove that p has at most $2^{|A|+|T|}$ non-forking global extensions.

Problem 5. Show that in DLO both the symmetry and transitivity of non-forking fail.