Stable groups, List 4

Problem 1. Let $(G, +)$ be an infinite abelian group of exponent 2 (so is is naturally a linear space over $\mathbb{Z}_2$). Choose an infinite linearly independent subset $A \subset G$. For $n \geq 1$ define A_n as the set of the sums of n pairwise distinct elements A, and moreover put $A_0 := \{0\}$.
(a) Prove that the structure $(G, +, A_n)_{n\in\omega}$ has quantifier elimination.
(b) Describe types in $S_1(G)$, namely show that:
(i) there is a unique type $p \in S_1(G)$ which contains all formulas $\neg A_n(x - a)$, $n \in \omega$, $a \in G$,
(ii) for any other type $q \in S_1(G)$ there is a smallest $n \in \omega$ such for some $a \in G$ the formula $A_n(x - a)$ belongs to q, and that n and a determine q.
(c) Show that $(G, +, A_n)_{n\in\omega}$ is ω-stable and $RM(G) = \omega$. Notice that p is a generic type of G.
(d) Prove that each proper, definable subgroup of G is finite.
(e) Using A, observe that the assumption of the finiteness of RM is needed in ZIT.

Problem 2. Let G be an ω-stable group. Prove that if $A, B \subseteq G$ are such that $G \setminus A$ i $G \setminus B$ are not generic, then $G = A \cdot B$.

Problem 3. Let G be a group of finite Morley rank. Assume $A \subseteq G$ is indecomposable. Prove that $\langle A^{-1}A\rangle$ is definable and connected.

Problem 4. Let G be a stable group, and S a definable group of automorphisms of G (i.e. both the group S and the action of S on G are definable in the stable structure in which we are working). Assume that a definable $A \subseteq G$ is S-invariant (i.e. $sA = A$ for every $s \in S$). Prove that A is indecomposable iff for every definable S-invariant subgroup $H < G$, if $|A/H| > 1$, then $|A/H| \geq \omega$.

Problem 5. Prove that each field of finite Morley rank is $\aleph_1$-categorical and quasi-strongly minimal.