Stable groups, List 6

Problem 1. Let $M \models T$, and $\theta(\bar{x}) \in \mathrm{tp}(\bar{c}/A\psi(M))$ be a formula isolating $\mathrm{tp}(\bar{c}/A\psi(M))$, for some $A, \bar{c} \subseteq M$ and $\psi(\bar{y}) \in L(M)$. Prove that for every $N \succ M$ the formula $\theta(\bar{x})$ also isolates $\mathrm{tp}(\bar{c}/A\psi(N))$.

Problem 2. In a stable theory, suppose that a sequence (a_i) is independent over $B \supseteq A$ and each $a_i \mathop{\smile\kern-0.9em\mid}_A B$. Prove that the sequence (a_i) is independent over A and is also independent from B over A.

Problem 3. Let M be an $\aleph_1$-categorical structure, $\varphi(x)$ — a strongly minimal formula (without parameters), and $a \in M \setminus \mathrm{acl}(\emptyset)$. Prove that $\mathrm{tp}(a)$ is not foreign to $\varphi(x)$.

Hint. Show that otherwise there is a Vaught pair for $\varphi(x)$.

Problem 4. Assume that K is an ω-stable field or an ω-stable definably simple group. Let $q = \mathrm{tp}(b/K)$ be non-orthogonal to a generic type of K. Deduce from Hrushovski's theorem (the one whose proved we skipped) that K is q-internal.

Problem 5*. Let $G = (\mathbb{Z}_4^\omega, +)$. Prove that G is $\aleph_1$-categorical, $2G$ and $G/2G$ are strongly minimal, but G is not quasi-strongly minimal.

Comment. This shows that the assumption on non-existence of an infinite interpretable group is essential in Zilber's theorem.