Topological dynamics in model theory. List 12.

Let M be a model and $\mathfrak{C} \succ M$ a monster model of a complete theory T.

Problem 1. For the structure M_n from Example 4 on page 74, show that $\mathrm{Aut}(M_n)$ acts transitively on the family of all open, short intervals.

Problem 2. Prove that the relation on $\prod_{n>1} \mathbb{Z}/n\mathbb{Z} = \prod_n((-n/2, n/2] \cap \mathbb{Z})$ of lying in the same coset modulo the subgroup of all bounded sequences is $\sim_B l^\infty$.

Problem 3. Let S and Q be products of sorts, and $X \subseteq S$, $Y \subseteq Q$ be $\emptyset$-type-definable. Let $\bar{x}$, $\bar{y}$ be tuples of variables corresponding to S and Q, respectively.

(i) Prove that for every finite tuples $\bar{x}_0, \bar{x}_1 \subseteq \bar{x}$ of the same length (meaning also from the same sorts), for every $p, q \in S_{\bar{x}}(\mathfrak{C})$ and for every $\eta \in E(S_{\bar{x}}(\mathfrak{C}))$, if $p\restriction_{\bar{x}_0} = q\restriction_{\bar{x}_1}$, then $\eta(p)\restriction_{\bar{x}_0} = \eta(q)\restriction_{\bar{x}_1}$ (after the obvious identification of variables).

(ii) Prove that under the assumptions of the lemma on page 77, the function $f\colon E(S_X(\mathfrak{C})) \to E(S_Y(\mathfrak{C}))$ from the proof of this lemma really takes values in $E(S_Y(\mathfrak{C}))$ and is a semigroup and $\mathrm{Aut}(\mathfrak{C})$-flow isomorphism.

Problem 4. Assume T is stable. Let $S(\emptyset) \ni p_0 \subseteq p \in S(M)$. Prove that p does not fork over $\emptyset$ if and only if p is minimal in the fundamental order on the set $S_{p_0}(M)$.

Problem 5. Let $p, q \in S(M)$.

(i) Prove that $q \leq^c p \Longrightarrow q \leq p$.

(ii) Assume T is stable and M is $\aleph_0$-saturated. Prove that $q \leq^c p \iff q \leq p$.

Problem 6. Assume M is $\aleph_0$-saturated and $A \supseteq M$.

(i) Prove that every type $p \in S(M)$ has an extension $p' \in S(A)$ which is a strong coheir over M.

(ii) Prove that every type $p \in S(M)$ has an extension $p' \in S(A)$ which is a strong heir over M.