

Topological dynamics in model theory. List 13.

Problem 1. Let M be any model. Let \mathcal{M} be a minimal left ideal in $S(M)$ and $u \in \mathcal{M}$ an idempotent. Let $\bar{J} = \text{Im}(u)$ be equipped with the induced infinitary patterns structure. Using Ellis theorem, prove that \bar{J} does not depend on the choice of \mathcal{M} and u .

Problem 2. Prove the equivalences (1) \leftrightarrow (2) and (1) \leftrightarrow (3) in Proposition 10 on page 82.

Problem 3. Let M be a strongly \aleph_0 -homogeneous model. Let $Q \subseteq S(M)$. Prove that Q is ip-minimal if and only if every finite subset of Q is ip-minimal.

Problem 4. Let M be a strongly \aleph_0 -homogeneous model. Prove that an ip-minimal subset Q of $S(M)$ which is an image of $S(M)$ by some morphism is isomorphic to \bar{J} from Problem 1.

Problem 5. Prove in details Corollary 18 on page 84, including the fact that the Ellis group remains the same as a topological group.

Comment. You can use here Fact 5 without a proof and the definition of the *ipp-topology* from page 80. But you can, of course, prove Fact 5 as well.