Topological dynamics in model theory. List 6.

In Problems 2-5, M denotes an o-minimal expansion of $(\mathbb{R}, +, \cdot, \leq)$, and $\mathfrak{C} \succ M$ is a monster model.

Problem 1. Let $G = G(M)$ be a group definable in a model M. Let $N \succ M$. Prove that an heir extension of a G-invairnt type $p \in S_G(M)$ to a complete type over N is $G(N)$-invariant.

Problem 2. Suppose $\mathrm{tp}(\bar{b}/\mathbb{R}, \bar{a})$ is an heir over $\mathbb{R}$. Let X and Y be $\mathbb{R}$-definable subsets of $\mathfrak{C}$ (i.e. of finite Cartesian powers of $\mathfrak{C}$) contained in the ball of radius $r \in \mathbb{R}$ centered at 0, and let $f\colon X \to Y$ be a continuoaus, $\bar{a}$-definable function. Prove that $\mathrm{st}(f(\bar{b})) = \mathrm{st}(f(\mathrm{st}(\bar{b})))$.

Problem 3. Let G be a definable group in M with a definable compact-torsion-free decomposition KH. Prove that the function $\psi\colon K \to K$ defined at the bottom of page 39 of the lecture notes is definable.

Problem 4. Let G be a definable group in M with a definable compact-torsion-free decomposition KH. Prove that if $Z(G)$ is finite, then $Z(G)$ embeds into the Ellis group of the G-flow $S_G(M)$.

Problem 5. Prove that for $G := \mathrm{SL}_n(\mathbb{R})$, $K := \mathrm{SO}_n(\mathbb{R})$, and $H := \mathrm{T}_n^+(\mathbb{R})$, $F := K \cap N_G(H) \cong \mathbb{Z}_2^{n-1}$. (Hence, the Ellis group of the G-flow $S_G(M)$ is $\mathbb{Z}_2^{n-1}$.)