Measures on minimally generated Boolean algebras

Piotr Borodulin-Nadzieja

Athens, July 2005

QUESTIONS

Efimov Problem Does exist an infinite compact Hausdorff space without nontrivial converging sequences which does not contain any copy of $\beta\omega$?

Remark Compact space contains a copy of $\beta\omega$ if and only if it can be mapped continuously onto $[0,1]^c$.

Answers
Fedorčuk (1975):
$\diamondsuit \Rightarrow$ YES.
Alan Dow (2004):
$(2^s < 2^c$ and $cof([s]^\omega, \subseteq) = s) \Rightarrow$ YES.

Weak Efimov Problem Does exist an infinite compact Hausdorff space without nontrivial converging sequences which carries only separable Radon measures?

Answer
Plebanek & Džamonja (2005): $CH \Rightarrow$ YES.

Problem How to characterize topologically compact spaces which carries only separable Radon measures?

Answer
Fremlin (1996): Under $MA(\omega_1)$ a space X carries nonseparable measure if and only if there is a continuous function from X onto $[0,1]^{\omega_1}$.

DEFINITIONS

Definition We will say that Boolean algebra B extends algebra A minimally if there is no proper subalgebra of B containing A.

Definition Boolean algebra B is minimally generated over A if there is a continuous sequence of algebras $(A_\alpha)_{\alpha<\kappa}$, such that $A_0 = A$, $A_{\alpha+1}$ is minimal extension of A_α for every $\alpha < \kappa$ and $A_\kappa = B$.

Definition Boolean algebra is minimally generated (m.g.) if it is minimally generated over $\{0, 1\}$.

Reference Sabine Koppelberg , *Minimally Generated Boolean Algebras* (1989)

PROPERTIES

- length of m.g. Boolean alebra does not need to be a cardinal number but it has to be limit ordinal;

- property of being a m.g. Boolean algebra of sets is not closed under products and unions;

- every measure on m.g. Boolean algebra is separable;

- every nonatomic measure on m.g. Boolean algebra generated in ω_1 steps is uniformly regular;

- none of above implications can be reversed;

- if A is m.g. Boolean algebra then the space $Stone$(A) does not contain any copy of $\beta\omega$.

EXAMPLE

Let A be a Boolean algebra minimally generated over $FIN(\omega)$ such that there is no minimal extension of A. Denote by X the space $Stone$(A).

Remark For every nontrivial sequence of natural numbers we can find a set $A \in$ A such that both A and A^c contains infinitely many elements of the sequence.

Corollary The space X does not contain a copy of $\beta\omega$ but no nontrivial sequence of open sets in X converges to point.