Berkovich spaces, Problem List 6

Let $(k, |\cdot|)$ be a non-Archimedean complete algebraically closed field.

1. Show that the set $\{\sum a_i T^i \mid \sup |a_i| r^i < \infty\}$ is a k-subalgebra of $k[\![T]\!]$.

2. Show that there is an isomorphism

$$\left\{ \sum_{i=0}^{\infty} a_i T^i \mid \sup |a_i| r^i < \infty \right\} \cong k\{r^{-1}T\}$$

   of normed $k[T]$-algebras.

3. Let R be a normed ring. Show that the restriction map

$$\mathcal{M}(\widehat{R}) \to \mathcal{M}(R)$$

   is a bijection.

4. Show that the bijection from the lecture

$$\psi_r : \mathcal{M}(K\{r^{-1}T\}) \to \mathcal{U}_r$$

   is a homeomorphism.

5. Let $\mathcal{A}, \mathcal{B}$ be Banach rings and $f : \mathcal{A} \to \mathcal{B}$ be admissible epimorphism. Show that f is bounded.

6. Let $V \subseteq \mathbb{A}_k^n$ be an affine algebraic variety. Show that the image of the natural map

$$V^{\mathrm{an}} \to \mathbb{A}^n_{\mathrm{Berk}}$$

   is closed.

7. Show that the completion of a perfect field is perfect.