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EMPIRICAL PROCESSES, VAPNIK-CHERVONENKIS CLASSES AND POISSON PROCESSES

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Abstract: For background of this paper see [2]. Given a probability space $(X, \mathcal{A}, P)$, let G_P be the Gaussian process with mean 0, indexed by $\mathcal{A}$, and such that

$$EG_P(A)G_P(B) = P(A \cap B) - P(A)P(B),\ A, B \in \mathcal{A}.$$

(1) Let $\mathcal{C} \subset \mathcal{A}$ and suppose that, for all probability measures, (laws) Q on $\mathcal{A}$, G_Q has a version with bounded sample functions on $\mathcal{C}$. (For example, suppose $\mathcal{C}$ is a "universal Donsker class".) Then, for some: n, no set F of n elements has all its subsets of the form $C \cap F, C \in \mathcal{C}$, i.e. $\mathcal{C}$ is a Vapnik-Chervonenkis class. An example shows that limit theorems for empirical measures need not hold uniformly over a Vapnik-Chervonenkis class of measurable sets, unless further measurability is assumed.
(2) For a law P on $X = \{1, 2, ...\}$, the collection 2^X of all subsets is a Donsker class if and only if

$$\sum_m P(m)^{1/2} < \infty.$$

(3) For any probability space $(X, \mathcal{A}, P)$, suppose $\mathcal{C}$ is a P-Donsker class, $\mathcal{C} \in \mathcal{A}$. Let T_a be a Poisson point process with intensity measure aP, $a > 0$. Then, as $a \to \infty$, $(T_a - aP)/a^{1/2}$ converges in law, with respect to uniform convergence on $\mathcal{C}$, to the Gaussian process W_P with mean 0 and $EW_P(A)W_P(B) = P(A \cap B),\ A, B \in \mathcal{C}$.

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