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REMARKS ON THE POSNTVTTY OF DENSITIES OF STABLE LAWS

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Abstract: Let $0 < \alpha < \infty$, $\alpha \neq 1$, and $\mathcal{S}$ be a non-empty subset of R^d, the d-dimensional Euclidean space. It is shown that if $\mathcal{S}$ satisfies $a\mathcal{S} + b\mathcal{S} = \mathcal{S}$ whenever $a, b \geq 0$ with $a^\alpha + b^\alpha = 1$, then $\mathcal{S}$ is a convex cone with vertex at 0. This, in particular, confirms a conjecture of Port and Vitale [4]. Using this result, an elementary, completely geometric and unified proof is provided for the following known result concerning, the positivity properties of densities of α-stable laws on R^d, $0 < \alpha < 2$, $\alpha \neq 1$: Let X be a strictly α-stable random vector in R^d with truly d-dimensional law μ, and let $p(t, \cdot)$ and σ be the density of $t^{1/\alpha}\mu$, the law $t^{1/\alpha}X$, and the spectral measure of μ, respectively. If $0 < \alpha < 1$ and the support of σ is contained in a half-space, then, for any $t > 0$, $p(t, x) > 0$ if and only if x belongs to the interior of the convex cone generated by support of σ; and, in all other cases, $p(t, x) > 0$ for all $t > 0$ and $x \in R^d$.

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