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A CHARACTERIZATION OF THE BIVARIATE WISHART DISTRIBUTION

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Abstract: We provide a characterization of the bivariate Wishart and normal-Wishart distributions. Assume that $\vec{x} = \{x_1, x_2\}$ has a non-singular bivariate normal pdf $f(\vec{x}) = N(\vec{\mu}, W)$ with unknown mean vector $\vec{\mu}$ and unknown precision matrix W. Let $f(\vec{x}) = f(x_1)f(x_2|x_1)$, where $f(x_1) = N(m_1, 1/v_1)$ and $f(x_2|x_1) = N(m_{2|1} + b_{12}x_1, 1/v_{2|1})$. Similarly, define $\{v_2, v_{1|2}, b_{21}, m_2, m_{1|2}\}$ using the factorization $f(\vec{x}) = f(x_2)f(x_1|x_2)$. Assume $\vec{\mu}$ and W have a strictly positive joint pdf $f_{\vec{\mu}W}(\vec{\mu}, W)$. Then $f_{\vec{\mu},W}$ is a normal-Wishart pdf if and only if global independence holds, namely,

$$\{v_1, m_1\} \perp \{v_{2|1}, b_{12}, m_{2|1}\} \quad \text{and} \quad \{v_2, m_2\} \perp \{v_{1|2}, b_{21}, m_{1|2}\},$$

and local independence holds, namely,

$$\perp \{v_1^*, m_1^*\}, \perp \{v_{2|1}^*, b_{12}^*, m_{2|1}^*\} \quad \text{and} \quad \perp \{v_2^*, m_2^*\}, \perp \{v_{1|2}^*, b_{21}^*, m_{1|2}^*\},$$

(where x^* denotes the standardized r.v. x and $\perp$ stands for independence). We also characterize the bivariate pdfs that satisfy global independence alone. Such pdfs are termed hyper-Markov laws and they are used for a decomposable prior-to-posterior analysis of Bayesian networks.

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