ON THE DUGUÉ PROBLEM WITH A SOLUTION IN THE SET OF SIGNED MEASURES

Marek T. Malinowski
Jolanta K. Misiewicz

Abstract: There are two methods of obtaining symmetric probability measure on a base of an arbitrary probability measure μ corresponding to the random variable X. The first relies on considering distribution of $Y = X - X'$, where X' is an independent copy of X. In the language of measures we have then $\mathcal{L}(Y) = \mu * \mu^-$, where $\mu^-(A) = \mu(-A)$. In the second method we consider the mean of two measures μ and μ^-. In the paper we want to present some known and new results on characterizing such measures μ for which both methods coincide, i.e. measures for which

$$\frac{1}{2}(\mu + \mu^-) = \mu * \mu^-.$$

In the literature one can find also the following generalization of this question: for fixed $p \in (0, 1]$ what is the characterization of such pairs of distributions μ and ν for which

$$p\mu + (1 - p)\nu = \mu * \nu?$$

This problem was posed by Dugué in 1939 and it was extensively studied since then. However, the full characterization has not been found yet. In the paper we show some constructions of the Dugué question with the properties of simple fractions classes of characteristic functions. We give also a collection of new solutions and an example of three measures μ, ν and η such that

$$p\mu + q\nu + r\eta = \mu * \nu * \eta.$$

In the last section we give also some solutions in the set of signed σ-finite measures. The authors would like to express their gratitude to Professor D. Szynal for his interesting questions and discussions.

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