

WEAK-TYPE INEQUALITY FOR THE MARTINGALE SQUARE
FUNCTION AND A RELATED CHARACTERIZATION OF HILBERT
SPACES

Adam Osękowski

Abstract: Let f be a martingale taking values in a Banach space \mathcal{B} and let $S(f)$ be its square function. We show that if \mathcal{B} is a Hilbert space, then

$$\mathbb{P}(S(f) \geq 1) \leq \sqrt{e}\|f\|_1$$

and the constant \sqrt{e} is the best possible. This extends the result of Cox, who established this bound in the real case. Next, we show that this inequality characterizes Hilbert spaces in the following sense: if \mathcal{B} is not a Hilbert space, then there is a martingale f for which the above weak-type estimate does not hold.

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