PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 32, Fasc. 2 (2012), pp. 227–239

MOMENTS OF POISSON STOCHASTIC INTEGRALS WITH RANDOM INTEGRANDS

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Abstract: We show that the moment of order n of the Poisson stochastic integral of a random process $(u_x)_{x\in X}$ over a metric space X is given by the non-linear Mecke identity

$$E\big[\big(\int_X u_x(\omega)\omega(dx)\big)^n\big] = \sum_{\{P_1,\ldots,P_k\}\in\mathcal{P}_n} E\big[\int_{X^k} \varepsilon^+_{\mathfrak{s}_k}(u_{s_1}^{|P_1|}\ldots u_{s_k}^{|P_k|})\sigma(ds_1)\ldots\sigma(ds_k)\big],$$

where the sum runs over all partitions $P_1\cup\ldots\cup P_k$ of $\{1,\ldots,n\}$, $|P_i|$ denotes the cardinality of P_i, and $\varepsilon^+_{\mathfrak{s}_k}$ is the operator that acts by addition of points at $s_1,\ldots,s_k$ to Poisson configurations. This formula recovers known results in case $\big(u(x)\big)_{x\in X}$ is a deterministic function on X.

2000 AMS Mathematics Subject Classification: Primary: 60G57; Secondary: 60G55, 60H07.

Keywords and phrases: Poisson stochastic integrals, moment identities, Bell polynomials, Poisson–Skorohod integral.