

MULTIDIMENSIONAL CATALAN AND RELATED NUMBERS AS
HAUSDORFF MOMENTS

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Abstract: We study integral representation of the so-called d -dimensional Catalan numbers $C_d(n)$, defined by $[\prod_{p=0}^{d-1} p!/(n+p)!](dn)!$, $d = 2, 3, \dots$, $n = 0, 1, \dots$. We prove that the $C_d(n)$'s are the n th Hausdorff power moments of positive functions $W_d(x)$ defined on $x \in [0, d^d]$. We construct exact and explicit forms of $W_d(x)$ and demonstrate that they can be expressed as combinations of $d-1$ hypergeometric functions of type ${}_{d-1}F_{d-2}$ of argument x/d^d . These solutions are unique. We analyze them analytically and graphically. A combinatorially relevant, specific extension of $C_d(n)$ for d even in the form

$$D_d(n) = \left[\prod_{p=0}^{d-1} \frac{p!}{(n+p)!} \right] \left[\prod_{q=0}^{d/2-1} \frac{(2n+2q)!}{(2q)!} \right]$$

is analyzed along the same lines.

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