PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 36, Fasc. 2 (2016), pp. 353–368

A FUNCTIONAL LIMIT THEOREM FOR LOCALLY PERTURBED RANDOM WALKS

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Abstract: A particle moves randomly over the integer points of the real line. Jumps of the particle outside the membrane (a fixed "locally perturbating set") are i.i.d., have zero mean and finite variance, whereas jumps of the particle from the membrane have other distributions with finite means which may be different for different points of the membrane; furthermore, these jumps are mutually independent and independent of the jumps outside the membrane. Assuming that the particle cannot jump over the membrane, we prove that the weak scaling limit of the particle position is a skew Brownian motion with parameter $\gamma \in [-1, 1]$. The path of a skew Brownian motion is obtained by taking each excursion of a reflected Brownian motion, independently of the others, positive with probability $2^{-1}(1 + \gamma)$ and negative with probability $2^{-1}(1 - \gamma)$. To prove the weak convergence result, we give a new approach which is based on the martingale characterization of a skew Brownian motion. Among others, this enables us to provide the explicit formula for the parameter γ. In the previous articles, the explicit formulae for the parameter have only been obtained under the assumption that outside the membrane the particle performs unit jumps.

2000 AMS Mathematics Subject Classification: Primary: 60F17; Secondary: 60G50.

Keywords and phrases: Functional limit theorem, locally perturbed random walk, martingale characterization, skew Brownian motion.