

THE LOCAL STRUCTURE OF q -GAUSSIAN PROCESSES

Włodzimierz Bryc
Yizao Wang

Abstract: The local structure of q -Ornstein–Uhlenbeck process and q -Brownian motion are investigated for all $q \in (-1, 1)$. These are classical Markov processes that arose from the study of noncommutative probability. These processes have discontinuous sample paths, and the local small jumps are characterized by tangent processes. It is shown that, for all $q \in (-1, 1)$, the tangent processes in the interior of the state space are scaled Cauchy processes possibly with drifts. The tangent processes at the boundary of the state space are also computed, but they are not well-known processes in classical probability theory. Instead, they can be associated with the free $1/2$ -stable law, a well-known distribution in free probability, via Biane’s construction.

2000 AMS Mathematics Subject Classification: Primary: 60G17, 60F17; Secondary: 60J35.

Keywords and phrases: q -Brownian motion, q -Ornstein–Uhlenbeck process, inhomogeneous Markov process, tangent process, self-similar process, Cauchy process, free stable law, Biane’s construction.

THE FULL TEXT IS AVAILABLE [HERE](#)