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WIENER PROCESSES AND STOCHASTIC INTEGRALS IN A BANACH SPACE

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Abstract. The representations of Wiener process by uniformly convergent series of one-dimensional Gaussian random processes in a separable Banach space are given (Section I). The Ito stochastic integral of an operator-valued random function by a Wiener process in a Banach space is defined (Section III); Section II contains an auxiliary material: there is defined a stochastic integral of a random function with values in the dual space.

The method of the paper is based on the use of the concept of covariance operator.

Let X be a real separable Banach space, X^* – its dual, $(\Omega, \mathscr{B}, P)$ – a probability space, and μ – a centered Gaussian measure on the Borel σ-algebra of X. It is well-known [16] that the characteristic functional of μ has the form

$$\hat{\mu}(x^*) = \exp\{-\tfrac{1}{2}\langle Rx^*, x^*\rangle\},$$

where $R\colon X^* \to X$ is a symmetric positive linear operator called the *covariance operator* of μ. Symmetric and positive linear operators $R\colon X^* \to X$, which are covariance operators of Gaussian measures, are called *Gaussian covariances*. A random element in X is called *Gaussian* if its distribution is a Gaussian measure.

I. Wiener processes. A family of random elements $(W_t)_{t\in[0,1]}$, $W_t\colon \Omega \to X$ is called a (*homogeneous*) *Wiener process* (with values in X) if

1. $W_0 = 0$ almost surely (a.s.);

2. for every $0 \leqslant t_0 < t_1 < \ldots < t_n \leqslant 1$, $W_{t_{i+1}} - W_{t_i}$ $(i = 0, \ldots, n-1)$ are independent random elements;

3. for every $t \in [0, 1]$, W_t is a centered Gaussian random element with covariance operator tR, where $R\colon X^* \to X$ is a fixed Gaussian covariance;

4. $(W_t)_{t\in[0,1]}$ has a.s. continuous sample paths.

If X is a finite-dimensional Hilbert space and R is the identity operator, then our definition of Wiener process coincides with the usual definition of finite-dimensional Wiener process. It is clear that if X is an infinite-dimensional Hilbert space, then there does not exist a Wiener process for which R is the identity operator. Our definition is a direct extension of the definition of a Wiener process for the Hilbert space case ([16], p. 113).

Gross [5] gave the definition of a Wiener process in the Banach space: using the concept of a measurable norm in a Hilbert space, he constructed a family of Gaussian measures $(\mu_t)_{t\in[0,1]}$; then, applying this family, he constructed a random process and verified the condition guaranteeing the continuity of this process. Here we propose another way of constructing the Wiener process in a Banach space.

Let $C([0, 1], X)$ be the vector space of all continuous functions from $[0, 1]$ into X. This is a separable Banach space with the norm

$$\|f\|_c = \operatorname*{Sup}_{t\in[0,1]} \|f(t)\|_X.$$

The functionals δ_{t,x^*}, $t\in[0, 1]$, $x^*\in X^*$, defined by $\langle f, \delta_{t,x^*}\rangle = \langle f(t), x^*\rangle$, separate points of $C([0, 1], X)$, i.e. the set $\Gamma \equiv \{\delta_{t,x^*}, t\in[0, 1], x^*\in X^*\}$ is a total subset of the dual Banach space $C([0, 1], X)^*$.

PROPOSITION 1.1. *Let $(W_t)_{t\in[0,1]}$ be a Wiener process in X. Then the random element $W: \Omega\to C([0, 1], X)$, defined by the equality $W(\omega)(t) = W_t(\omega)$, is a Gaussian random element in $C([0, 1], X)$.*

The covariance operator of W on the elements $\delta_{t,x^*}\in\Gamma$ takes the values $(R_W\delta_{t,x^*})(s) = \min(t, s)Rx^*$, where R is the covariance operator of W_1.

Conversely, if $W: \Omega\to C([0, 1], X)$ is a centered Gaussian random element with the convariance operator $(R_W\delta_{t,x^*})(s) = \min(t, s)Rx^*$, where R is a Gaussian covariance in X, then the random process $W_t(\omega) = W(\omega)(t)$ is a Wiener process in X, and the covariance operator of W_1 is R.

Proof. The measurability of the map $W: \Omega\to C([0, 1], X)$ follows from the continuity of sample paths of the Wiener process $(W_t)_{t\in[0,1]}$. The process $(W_t)_{t\in[0,1]}$ is Gaussian, i.e. for all $t_1,\ldots, t_n$ and $x_1^*, x_2^*,\ldots, x_n^*$,

$$\sum_{i=1}^{n} \langle W_{t_i}, x_i^*\rangle$$

is a Gaussian random variable. Therefore, for all φ^* from the linear span $L(\Gamma)$ of the total set Γ, $\langle W, \varphi^*\rangle$ is a Gaussian random variable, hence $\langle W, \varphi^*\rangle$ is Gaussian for all $\varphi^*\in C([0, 1], X)^*$ ([13], th. 11.), i.e. $W: \Omega \to C([0, 1], X)$ is Gaussian. The covariance operator R_W of the random element W transforms $C([0, 1], X)^*$ into $C([0, 1], X)$ and

$$\langle R_W\delta_{t,x^*}(s), y^*\rangle = \langle R_W\delta_{t,x^*}, \delta_{s,y^*}\rangle$$
$$= E(\langle W_t, x^*\rangle\langle W_s, y^*\rangle) = \min(t, s)\langle Rx^*, y^*\rangle.$$

Conversely, if $W\colon \Omega \to C([0, 1], X)$ is a centered Gaussian random element with the covariance operator $(R_W \delta_{t,x^*})(s) = \min(t, s) Rx^*$, then the random process $W_t(\omega) = W(\omega)(t)$, $\omega \in \Omega$, $t \in [0, 1]$, is a Wiener process in X. In fact, fulfilment of conditions 1, 3, and 4 of definition of the Wiener process immediately follows from the properties of the random element W. It is easy to verify that, for all n, $0 \leqslant t_1 < \ldots < t_n \leqslant 1$ and $x_1^*, \ldots, x_{n-1}^*$ from X^*, the random variables $\langle W_{t_{i+1}} - W_{t_i}, x_i^* \rangle$, $i = 1, \ldots, n-1$, are non-correlated. Hence the Gaussian random variables $\langle W_{t_{i+1}} - W_{t_i}, x_i^* \rangle$, $i = 1, \ldots, n-1$, are independent, i.e. the random elements $W_{t_{i+1}} - W_{t_i}$, $i = 1, \ldots, n-1$, are independent. Proposition is proved.

The following theorem is a generalization of the one-dimensional Ito-Nisio theorem ([6], th. 5.2). It shows also the existence of the Wiener process in a separable Banach space for every Gaussian covariance R.

THEOREM 1.1. *Let $(e_n)_{n \in N}$ be an orthonormal basis in $L_2[0, 1]$, and $(\xi_n)_{n \in N}$ – a sequence of independent identically distributed centered Gaussian random elements in X. Then the series*

$$\sum_{n=1}^{\infty} \int_0^t e_n(\tau)\, d\tau\, \xi_n(\omega) \equiv W_t$$

a.s. uniformly in t converges in X, and the sum $(W_t)_{t \in [0,1]}$ is a Wiener process in X.

The covariance operator of W_t is tR, where R is the covariance operator of ξ_1.

Proof. We first show that, for some $c > 0$,

$$P\left\{\limsup_{n \to \infty} \left\{\frac{\|\xi_n\|}{\sqrt{\log n}} > c\right\}\right\} = 0.$$

It follows from the Chebyshev inequality that

$$P\left\{\omega\colon \frac{\|\xi_n\|}{(\log n)^{1/2}} > c\right\} \leqslant \frac{E \exp(\alpha \|\xi_n\|^2)}{e^{\alpha c^2 (\log n)}} = \frac{E \exp(\alpha \|\xi_n\|^2)}{n^{\alpha c^2}}$$

for all positive constants α and c. The Fernique theorem [4] implies the existence of an $\alpha > 0$ such that

$$E \exp(\alpha \|\xi_n\|^2) \equiv c_1 < \infty \qquad \text{for all } n = 1, 2, \ldots$$

Choose the constant $c > 0$ such that $\alpha c^2 > 1$. Then

$$P\left\{\omega\colon \frac{\|\xi_n\|}{(\log n)} \geqslant c\right\} \leqslant \frac{c_1}{n^{\alpha c^2}}$$

and the series

$$\sum_{n=1}^{\infty} P\left\{\omega\colon \frac{\|\xi_n\|}{\sqrt{\log n}} > c\right\}$$

is convergent. Hence, by the Borel–Cantelli lemma, we have

$$P\left\{\limsup_{n\to\infty}\left\{\frac{\|\xi_n\|}{\sqrt{\log n}}>c\right\}\right\}=0,$$

i.e.

$$\|\xi_n\|=O(\sqrt{\log n}),\ n=1,2,\ldots \text{ a.s.}$$

Let now $(e_k)_{k\in N}$ be the Haar orthonormal basis in $L_2[0, 1]$. It is well-known (see e.g. [8], § 21) that the number series

$$\sum_{k=1}^{\infty} a_k \int_0^t e_k(\tau)\,d\tau$$

is uniformly in t absolutely convergent if $\|a_k\|=O(k^{\varepsilon})$ for any $\varepsilon<\frac{1}{2}$. Therefore the estimation of tails of $\|\xi_n\|$, $n=1,2,\ldots$, gives the convergence of the series

$$\sum_{k=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\,\xi_k$$

a.s. uniformly in t if $(e_k)_{k\in N}$ is the Haar basis. It is evident that the sum will be an X-valued function a.s. continuous in t, i.e. it represents a random element in $C([0, 1], X)$. The random element $W\colon \Omega\to C([0, 1], X)$,

$$W(\omega)(t)=\sum_{k=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\,\xi_k,\qquad t\in[0, 1],\ \omega\in\Omega,$$

is Gaussian with zero mean and

$$\langle R_W\delta_{t,x^*},\delta_{s,y^*}\rangle=E\Big(\sum_{k=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\,\langle\xi_k,x^*\rangle\Big)\Big(\sum_{k=1}^{\infty}\int_0^s e_k(\tau)\,d\tau\,\langle\xi_k,y^*\rangle\Big)$$

$$=\sum_{k=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\int_0^s e_k(\tau)\,d\tau E\,\langle\xi_k,x^*\rangle\langle\xi_k,y^*\rangle=\min(t,s)\,\langle Rx^*,y^*\rangle.$$

Therefore, the sum

$$\sum_{k=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\,\xi_k$$

is a Wiener process. The existence of the Wiener process is proved.

Let now $(e_n)_{n\in N}$ be an arbitrary orthonormal basis in $L_2[0, 1]$. It is clear that $\zeta_k\colon \Omega\to C([0, 1], X)$,

$$\zeta_k(\omega)(t)=\int_0^t e_k(\tau)\,d\tau\,\xi_k(\omega),\qquad k=1,2,\ldots,$$

are independent symmetric random elements in $C([0, 1], X)$. Let

$$S_n\colon \Omega\to C([0, 1], X),\ s_n=\sum_{k=1}^{n}\zeta_k,\qquad n=1,2,\ldots,$$

be a sequence of partial sums, μ_n be the distribution of the Gaussian random elements S_n, and $\hat{\mu}_n$ be the characteristic functional of the measure μ_n. Then

$$\hat{\mu}_n(\delta_{t,x^*}) = \exp\{-\tfrac{1}{2}\sum_{k=1}^{n}\big(\int_0^t e_k(\tau)\,d\tau\big)^2 \langle Rx^*, x^*\rangle\}$$

$$\to \exp\{-\tfrac{1}{2}t\,\langle Rx^*, x^*\rangle\} = \hat{\mu}_W(\delta_{t,x^*}),$$

where $\hat{\mu}_W$ is the characteristic functional of a measure on $C([0, 1], X)$ corresponding to a Wiener process (Wiener measure). It is easy to see that on the linear span of Γ we also have $\hat{\mu}_n \to \hat{\mu}_W$. Hence, by the theorem of Ito-Nisio ([6], th. 4.1), we have the a.s. convergence $(S_n)_{n\in N}$, i.e. the a.s. uniformly in t convergence of the series

$$\sum_{k=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\,\xi_k$$

to a Wiener process. Theorem is proved.

Now we shall construct another series of independent real-valued Wiener processes with coefficients from X, which will converge a.s. uniformly in t to the Wiener process in X. First we note that, by the factorization lemma ([16], p. 135), the symmetric and positive operator R: $X^* \to X$ can be factorized through the separable Hilbert space H: $R = AA^*$, where A: $H \to X$ is a continuous linear operator.

THEOREM 1.2. *Let $R = AA^*$ be a Gaussian covariance, A: $H \to X$, H be a separable Hilbert space. Then, for any orthonormal basis $(h_n)_{n\in N}$ in H and any sequence $(\zeta_n(t), t\in[0, 1])$ of independent real valued Wiener processes, the series*

$$\sum_{n=1}^{\infty} Ah_n\zeta_n(t) \equiv W_t, \qquad t\in[0, 1],$$

a.s. uniformly in t converges in X and the sum $(W_t)_{t\in[0,1]}$ is a Wiener process in X. The covariance operator of W_1 is R.

Proof. By Theorem 1.1, for any Gaussian covariance R there exists a Wiener process $(W_t)_{t\in[0,1]}$ in X, and we can consider the corresponding Gaussian random element W in $C([0, 1], X)$. Introduce a sequence of independent symmetric random elements

$$\eta_k\colon \Omega \to C([0, 1], X),\ \eta_k = Ah_k\zeta_k, \qquad k = 1, 2, \ldots$$

Let $S_n = \sum_{k=1}^{n}\eta_k$, μ_n be the distribution of S_n, $\hat{\mu}_n$ be the characteristic functional of μ_n. It is easy to see that, for $\delta_{t,x^*}\in\Gamma$,

$$\hat{\mu}_n(\delta_{t,x^*}) = \exp\big(-\tfrac{1}{2}t\sum_{k=1}^{n}\langle Ah_k, x^*\rangle^2\big)$$

and

$$\lim_{n\to\infty} \hat{\mu}_n(\delta_{t,x^*}) = \exp\left(-\tfrac{1}{2}t \sum_{k=1}^{\infty} (h_k, A^*x^*)_H^2\right)$$
$$= \exp\left(-\tfrac{1}{2}t \langle Rx^*, x^*\rangle\right) = \hat{\mu}_W(\delta_{t,x^*}),$$

where $(\cdot,\cdot)_H$ means the inner product in H. Since the convergence $\hat{\mu}_n \to \hat{\mu}_W$ takes place on the linear span of Γ and since Γ is total, we see, according to the Ito-Nisio theorem, that $(S_n)_{n\in N}$ converges a.s. in $C([0, 1], X)$, i.e. the series

$$\sum_{k=1}^{\infty} Ah_k \zeta_k(t)$$

converges a.s. uniformly in t in X. It is clear that the limit process is a Wiener process in X. Theorem 1.2 is proved.

Remark. The mentioned results have been announced in our paper [10]. Chevet [1] obtained these results independently and practically simultaneously by a different method. These theorems have been considered also in [12] but the proof of the existence of Wiener process in [12] is not correct.

Let H be a separable Hilbert space. Define by $L_2([0,1], H)$ the separable Hilbert space of vector functions $\varphi:[0, 1] \to H$ for which

$$\int_0^1 \|\varphi(t)\|_H^2 \, dt < \infty.$$

LEMMA 1.1. *Let μ_W be a Wiener measure on $C([0, 1], X)$ with the covariance operator $(R_W \delta_{t,x^*})(s) = \min(t, s) Rx^*$, $\delta_{t,x^*} \in \Gamma$, where R: $X^* \to X$ is a Gaussian covariance in X, and let $AA^* = R$ be its factorization through a separable Hilbert space $H (A$: $H \to X)$. Then the operator*

$$T\colon L_2([0, 1], H) \to C([0, 1], X), \; Th(t) = \int_0^t A\big(h(\tau)\big)\, d\tau$$

transforms the canonical Gaussian cylindrical measure on $L_2([0, 1], H)$ into μ_W on $C([0, 1], X)$.

Proof. It is sufficient (see e.g. [2]) to proof the coincidence of the operators TT^*: $C([0, 1], X)^* \to C([0, 1], X)$ and R_W. To this end it suffices to prove that they coincide on the total subset $\Gamma \subset C([0, 1], X)^*$. So we have to verify that

$$\langle TT^* \delta_{t,x^*}, \delta_{s,y^*}\rangle = \langle R_W \delta_{t,x^*}, \delta_{s,y^*}\rangle \quad \text{for all } t, s \in [0, 1] \text{ and } x^*, y^* \in X^*.$$

Let $h \in L_2([0, 1], H)$. We shall calculate $(T^* \delta_{t,x^*}, h)_{L_2(H)}$, where

$(\cdot, \cdot)_{L_2(H)}$ means the inner product in $L_2([0, 1], H)$. We have

$$\begin{aligned}(T^* \delta_{t,x^*}, h)_{L_2(H)} &= \langle Th, \delta_{t,x^*}\rangle \\ &= \int_0^t \langle Ah(\tau), x^*\rangle d\tau = \int_0^t (h(\tau), A^*x^*)_H d\tau \\ &= (\chi_{[0,t]} A^*x^*, h)_{L_2(H)},\end{aligned}$$

where $\chi_{[0,t]}$ is the indicator of the set $[0, t]$, i.e.

$$\chi_{[0,t]}(\tau) = \begin{cases} 1, & \tau \in [0, t], \\ 0, & \tau \notin [0, t]. \end{cases}$$

Consequently, $T^* \delta_{t,x^*} = \chi_{[0,t]} A^* x^*$. Then

$$\begin{aligned}\langle TT^* \delta_{t,x^*}, \delta_{s,y^*}\rangle &= \langle \chi_{[0,t]} A^* x^*, \chi_{[0,s]} A^* y^*\rangle \\ &= \min(t, s) \langle Rx^*, y^*\rangle,\end{aligned}$$

i.e. $TT^* = R_W$. Lemma 1.1 is proved.

Now we proof that all Wiener processes have the representation established in Theorem 1.1.

THEOREM 1.3. *Let $(W_t)_{t\in[0,1]}$ be an arbitrary Wiener process in X, and $(e_n)_{n\in N}$ – an orthonormal basis in $L_2[0, 1]$. Then there exists a sequence $(\xi_n)_{n\in N}$ of independent identically distributed centered Gaussian random elements in X such that*

$$W_t(\omega) = \sum_{k=1}^{\infty} \int_0^t e_k(\tau)\, d\tau\, \xi_k(\omega) \quad \textit{a.s.}$$

Proof. Let $(W_t)_{t\in[0,1]}$ be an arbitrary Wiener process and let the covariance operator of W_1 be $R = AA^*$. Since the operator

$$T\colon L_2([0, 1], H) \to C([0, 1], X), \qquad Th(t) = \int_0^t Ah(\tau)\, d\tau$$

has the property $TT^* = R_W$, for all orthonormal bases $(f_k)_{k\in N}$ from the Hilbert space $L_2([0, 1], H)$ there exists a sequence of standard independent Gaussian random variables $(\gamma_k)_{k\in N}$ such that

$$W = \sum_{k=1}^{\infty} Tf_k \gamma_k \quad \text{a.s.}$$

and the convergence is meant in $C([0, 1], X)$ (see e.g. [11]). Let $(h_k)_{k\in N}$ be an orthonormal basis in H, and $(e_k)_{k\in N}$ – an orthonormal basis in $L_2[0, 1]$. Then $(e_k h_j)_{k,j\in N}$ is an orthonormal basis in $L_2([0, 1], H)$. Therefore

$$W_t = \sum_{k=1}^{\infty} TQ_{\varphi^{-1}(k)} \gamma_{\varphi^{-1}(k)} \quad \text{a.s.},$$

where $\varphi\colon N^2 \to N$ is some ordering, $Q_{\varphi^{-1}(k)}$ is an element of the basis $(e_k h_j)_{k,j\in N}$, $\gamma_{\varphi^{-1}(k)}$ is an element of the sequence $(\gamma_{kj})_{k,j\in N}$, i.e. we have the summation in the fixed order.

Take now any $x^* \in X^*$. Then

$$E\Big(\langle W_t, x^*\rangle - \sum_{k=1}^{n}\sum_{j=1}^{n}\int_0^t e_k(\tau)\,d\tau\,\langle Ah_j, x^*\rangle\gamma_{kj}\Big)^2$$

$$\leqslant \sum_{k=1}^{\infty}\sum_{j=n+1}^{\infty}\Big(\int_0^t e_k(\tau)\,d\tau\Big)^2\langle Ah_j, x^*\rangle^2 + \sum_{k=n+1}^{\infty}\sum_{j=1}^{\infty}\Big(\int_0^t e_k(\tau)\,d\tau\Big)^2\langle Ah_j, x^*\rangle^2$$

$$= t\sum_{j=n+1}^{\infty}\langle Ah_j, x^*\rangle^2 + \|A^*x^*\|_H \sum_{k=n+1}^{\infty}\Big(\int_0^t e_k(\tau)\,d\tau\Big)^2 \to 0, \qquad n\to\infty.$$

Consequently,

$$\sum_{k=1}^{n}\sum_{j=1}^{n}\int_0^t e_k(\tau)\,d\tau\,\langle Ah_j, x^*\rangle\gamma_{kj} \to \langle W_t, x^*\rangle \tag{1}$$

a.s. when $n\to\infty$. We have

$$\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\,Ah_j\gamma_{kj} = \sum_{k=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\,\xi_k,$$

where

$$\xi_k = \sum_{j=1}^{\infty} Ah_j\gamma_{kj}$$

are independent identically distributed Gaussian random elements with covariance operator $R = AA^*$. Therefore, by Theorem 1.1, the series

$$\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\,Ah_j\gamma_{kj}$$

is a.s. convergent in $C([0, 1], X)$. Hence, by formula (1), we immediately obtain that

$$\langle W_t, x^*\rangle = \sum_{k=1}^{\infty}\int_0^t e_k(\tau)\,d\tau\,\langle\xi_k, x^*\rangle$$

a.s. for all $t\in[0, 1]$ and $x^*\in X^*$. Therefore

$$W_t = \sum_{k=1}^{\infty}\int_0^{\infty} e_k(\tau)\,d\tau\,\xi_k \qquad \text{a.s.}$$

Theorem 1.3 is proved.

The following statement gives the representation of the Wiener process in the form of sum of one-dimensional independent Wiener processes.

THEOREM 1.4. *Let $(W_t)_{t\in[0,1]}$ be a Wiener process in X, the covariance* *operator of* W_1 *be* $R = AA^*$, A: $H \to X$, *H being a separable Hilbert space. Then, for any orthonormal basis* $(h_k)_{k\in N}$ *in H, there exists a sequence of independent real-valued Wiener processes* $(\zeta_k(t),\ t\in[0, 1])_{k\in N}$ *such that*

$$W_t = \sum_{k=1}^{\infty} Ah_k\,\zeta_k(t) \qquad a.s.$$

We omit the proof of this theorem; it is quite similar to that of Theorem 1.3.

II. Stochastic integral of random function with values in the dual space. Let $(W_t)_{t\in[0,1]}$ be a Wiener process in X and suppose that on the probability space $(\Omega, \mathscr{B}, P)$ there is given a family $(\mathscr{F}_t)_{t\in[0,1]}$, $\mathscr{F}_t \subset \mathscr{B}$, of σ-algebras such that if $0 \leqslant t_1 < t_2 \leqslant 1$, then $\mathscr{F}_{t_1} \subset \mathscr{F}_{t_2}$ (in other words, $(\mathscr{F}_t)_{t\in[0,1]}$ is an increasing family). We say that $(\mathscr{F}_t)_{t\in[0,1]}$ is *adapted* to the Wiener process $(W_t)_{t\in[0,1]}$ if

(a) W_t is $\mathscr{F}_t$-measurable for all $t\in[0, 1]$;

(b) $W_s - W_t$ is independent of $\mathscr{F}_t$ for $0 \leqslant t < s \leqslant 1$.

As a typical example of an adapted family we can take the σ-algebras $\mathscr{F}_t$, $t\in[0, 1]$, generated by the random elements W_s, $0 \leqslant s \leqslant t$.

We will repeatedly use the following

PROPOSITION 2.1. *Let* $(W_t)_{t\in[0,1]}$ *be a Wiener process, and* $(\mathscr{F}_t)_{t\in[0,1]}$ – *a family of* σ*-algebras adapted to* $(W_t)_{t\in[0,1]}$. *Then there exists a representation*

$$W_t = \sum_{k=1}^{\infty} Ah_k\,\zeta_k(t)$$

such that $(\mathscr{F}_t)_{t\in[0,1]}$ *is adapted to the real-valued Wiener process* $(\zeta_k(t))_{t\in[0,1]}$ *for all* $k\in N$.

Proof. Let R: $X^* \to X$ be the covariance operator of W_1, $R = AA^*$, A: $H \to X$. There exist an orthonormal basis $(h_k)_{k\in N} \subset H$ and a sequence $(Q_k^*)_{k\in N} \subset X^*$ such that $\langle Ah_k, Q_j^*\rangle = \delta_{kj}$ for all $k, j\in N$ (see [17], p. 17). Let

$$W_t = \sum_{k=1}^{\infty} Ah_k\,\zeta_k(t)$$

be the corresponding representation of $(W_t)_{t\in[0,1]}$. Then the proof of the proposition follows from the equality

$$\langle W_t, Q_k^*\rangle = \sum_{i=1}^{\infty} \langle Ah_i\,\zeta_i(t), Q_k^*\rangle = \zeta_k(t).$$

Definition 2.1. A function φ: $[0, 1]\times\Omega \to X^*$ is called *non-anticipating with respect to* $(\mathscr{F}_t)_{t\in[0,1]}$ if

1. for all $x\in X$ the function $(t, \omega) \to \langle x, \varphi(t, \omega)\rangle$ from $([0, 1]\times\Omega, \mathscr{B}[0, 1]\times\mathscr{B})$ into $(R^1, \mathscr{B}(R^1))$ is measurable;

2. for all $x \in X$ and $t \in [0, 1]$ the function $\omega \to \langle x, \varphi(t, \omega)\rangle$ from $(\Omega, \mathscr{B})$ into $(R^1, \mathscr{B}(R^1))$ is $\mathscr{F}_t$-measurable.

Definition 2.2. We say that a non-anticipating function φ *belongs to the class* $G_R(X^*)$ if

$$p_R(\varphi) \equiv \Big(\int_0^1 \int_\Omega \langle R\varphi(t, \omega), \varphi(t, \omega)\rangle \, dt \, dP\Big)^{1/2} < \infty .$$

$G_R(X^*)$ is a linear space and p_R is a pseudonorm in it. Hence, we can introduce in $G_R(X^*)$ the topology generated by p_R. The linear topological spaces $(G_R(X^*), p_R)$ for different covariance operators R are different. If a family of σ-algebras $(\mathscr{F}_t)_{t\in[0,1]}$ is fixed and $R_1 \leqslant R_2$ are Gaussian covariances, then $G_{R_1}(X^*) \supset G_{R_2}(X^*)$. It is easy to see that, for an arbitrary covariance R, all non-anticipating functions from

$$L_2([0, 1] \times \Omega, \mathscr{B}[0, 1] \times \mathscr{B}, \lambda \times P, X^*)$$

are contained in $G_R(X^*)$ (λ denotes the Lebesgue measure on $[0, 1]$). Note also that if R is non-degenerate (i.e. $Rx^* = 0$ if and only if $x^* = 0$), then p_R is the norm in $G_R(X^*)$.

If $\varphi \in G_R(X^*)$ is a step-function,

$$\varphi(t, \omega) = \sum_{i=0}^{n-1} \varphi_{t_i}(\omega) \chi_{[t_i, t_{i+1}]},$$

$$0 = t_0 < \ldots < t_n = 1, \; \varphi_{t_i}\colon \Omega \to X^*, \; i = 0, \ldots, n-1,$$

then the stochastic integral of φ with respect to $(W_t)_{t\in[0,1]}$ is naturally defined by the equality

$$\int_0^1 \varphi \, dW(t) = \sum_{i=0}^{n-1} \langle \varphi_{t_i}, W_{t_{i+1}} - W_{t_i}\rangle .$$

LEMMA 2.1. *If $\varphi \in G_R(X^*)$ is a step-function, then*

$$E\Big(\int_0^1 \varphi \, dW(t)\Big) = 0 \quad \textit{and} \quad E\Big(\int_0^1 \varphi \, dW(t)\Big)^2 = \int_0^1 \int_\Omega \langle R\varphi, \varphi\rangle \, dt \, dP.$$

Proof. Let $\varphi = \sum_{i=0}^{n-1} \varphi_{t_i} \chi_{[t_i, t_{i+1}]}$. Then

$$E\Big(\int_0^1 \varphi \, dW(t)\Big)^2 = \sum_{i=0}^{n-1} E \langle \varphi_{t_i}, W_{t_{i+1}} - W_{t_i}\rangle^2 + \sum_{i \neq j = 1}^{n-1} E \langle \varphi_{t_i}, W_{t_{i+1}} - W_{t_i}\rangle \langle \varphi_{t_j}, W_{t_{j+1}} - W_{t_j}\rangle .$$

Let

$$W_t = \sum_{k=1}^{\infty} A h_k \zeta_k(t)$$

be a representation of the Wiener process such that $(\mathscr{F}_t)_{t\in[0,1]}$ is adapted to $(\zeta_k(t))_{t\in[0,1]}$ for all $k\in N$. For arbitrary $i\leqslant n-1$ we have

$$E\langle\varphi_{t_i}, W_{t_{i+1}}-W_{t_i}\rangle^2 = E\Big(\sum_{k=1}^{\infty}\langle Ah_k, \varphi_{t_i}\rangle\big(\zeta_k(t_{i+1})-\zeta_k(t_i)\big)\Big)^2$$

$$=(t_{i+1}-t_i)\sum_{k=1}^{\infty}E\langle Ah_k, \varphi_{t_i}\rangle^2=(t_{i+1}-t_i)E\Big(\sum_{k=1}^{\infty}\langle Ah_k, \varphi_{t_i}\rangle^2\Big)$$

$$=(t_{i+1}-t_i)\int_{\Omega}\langle R\varphi_{t_i}, \varphi_{t_i}\rangle dP.$$

It is easy to verify, that

$$\sum_{i\neq j=0}^{n-1}E\langle\varphi_{t_i}, W_{t_{i+1}}-W_{t_i}\rangle\langle\varphi_{t_j}, W_{t_{j+1}}-W_{t_j}\rangle=0.$$

Therefore

$$E\Big(\int_0^1\varphi\,dW(t)\Big)^2=\sum_{i=0}^{n-1}(t_{i+1}-t_i)\sum_{k=1}^{\infty}\langle\varphi_{t_i}, Ah_k\rangle^2$$

$$=\int_0^1\int_{\Omega}\langle R\varphi, \varphi\rangle\,dt\,dP.$$

Analogously we can show that

$$E\int_0^1\varphi\,dW(t)=0.$$

Lemma 2.1 is proved.

The following lemma will be used to define the stochastic integral of arbitrary $\varphi\in G_R(X^*)$.

LEMMA 2.2. *For an arbitrary $\varphi\in G_R(X^*)$ there exists a sequence of step-functions $(\varphi_n)_{n\in N}\subset G_R(X^*)$ such that $\varphi_n\xrightarrow{p_R} v$.*

Proof. Let $R=AA^*$, $A\colon H\to X$, be a factorization of R. Take $(h_k)_{k\in N}$, an orthonormal basis in H, and $(Q_k^*)_{k\in N}$ a sequence in X^* such that $\langle Ah_k, Q_j^*\rangle=\delta_{k,j}$ $(k, j\in N)$. Define

$$f_n=\sum_{k=1}^{n}\langle Ah_k, \varphi\rangle Q_k^*,\qquad n=1, 2, \ldots$$

We have $f_n\in G_R(X^*)$ and

$$p_R^2(f_n-\varphi)=\int_0^1\int_{\Omega}\langle R(f_n-\varphi), (f_n-\varphi)\rangle\,dt\,dP$$

$$=\int_0^1\int_{\Omega}\Big(\sum_{k=1}^{\infty}\langle Ah_k, \varphi-\sum_{j=1}^{n}\langle Ah_j, \varphi\rangle Q_j^*\Big)^2\,dt\,dP$$

$$=\int_0^1\int_{\Omega}\Big(\sum_{k=n+1}^{\infty}\langle Ah_k, \varphi\rangle^2\Big)\,dt\,dP\to 0,\qquad\text{when } n\to\infty.$$

For fixed $k \in N$, let $(\varphi_{km})_{m \in N}$ be a sequence of real-valued nonanticipating step-functions such that $\varphi_{km} \to \langle Ah_k, \varphi \rangle$ in $L_2(\Omega \times [0, 1], \mathscr{B} \times \mathscr{B}[0, 1], P \times \lambda)$, when $m \to \infty$. For fixed $n \in N$ let us define $(f_{nm})_{m \in N} \subset G_R(X^*)$ by

$$f_{nm} = \sum_{k=1}^{n} \varphi_{km} Q_k^*.$$

We have

$$p_R^2(f_{nm} - f_n) = \int_0^1 \int_\Omega \Big(\sum_{k=1}^{\infty} \langle Ah_k, \big(\sum_{j=1}^{n} (\varphi_{jm} Q_j^* - \langle \varphi, Ah_j \rangle Q_j^*) \big) \rangle^2 \Big) dt\, dP$$

$$= \sum_{k=1}^{n} \int_0^1 \int_\Omega (\varphi_{km} - \langle Ah_k, \varphi \rangle)^2 \, dt\, dP.$$

Therefore, for all $n \in N$, $f_{nm} \xrightarrow{p_R} f_n$, when $m \to \infty$. Hence, since $f_n \xrightarrow{p_R} f$, by virtue of a standard method we can choose a subsequence $(\varphi_n)_{n \in N}$ of $(f_{nm})_{n,m \in N}$ such that $p_R(\varphi_n - \varphi) \to 0$ when $n \to \infty$. Lemma 2.2 is proved.

Now, let $\varphi \in G_R(X^*)$. By Lemma 2.2, there exists a sequence of step-functions $(\varphi_n)_{n \in N} \subset G_R(X^*)$ such that $p_R(\varphi_n - \varphi) \to 0$ when $n \to \infty$. For arbitrary $n, m \in N$ we have

$$E\Big(\int_0^1 \varphi_n \, dW(t) - \int_0^1 \varphi_m \, dW(t)\Big)^2 = E\Big(\int_0^1 (\varphi_n - \varphi_m) \, dW(t)\Big)^2 = p_R^2(\varphi_n - \varphi_m).$$

Since $p_R(\varphi_n - \varphi_m) \to 0$ when $n, m \to \infty$, we infer that $\int_0^1 \varphi_n \, dW(t)$ converges in $L_2(\Omega, \mathscr{B}, P)$. Therefore we can define the stochastic integral for arbitrary $\varphi \in G_R(X^*)$.

Definition 2.3. Let $\varphi \in G_R(X^*)$. The limit in $L_2(\Omega, \mathscr{B}, P)$ of the sequence

$$\int_0^1 \varphi_n \, dW(t),$$

where $(\varphi_n)_{n \in N} \subset G_R(X^*)$ is an arbitrary sequence of step-functions converging to φ in pseudonorm p_R, is called the *stochastic integral of a random function* $\varphi \in G_R(X^*)$ *with respect to the Wiener process* $(W_t)_{t \in [0,1]}$.

The stochastic integral of φ is denoted by $\int_0^1 \varphi \, dW(t)$.

It is easy to see that the value of this limit does not depend of an approximating sequence of step-functions; in other words, the given definition of the stochastic integral is correct. Note also, that

$$E\Big(\int_0^1 \varphi \, dW(t)\Big)^2 = \int_0^1 \int_\Omega \langle R\varphi, \varphi \rangle \, dt\, dP = p_R^2(\varphi)$$

for any function $\varphi = G_R(X^*)$.

III. Stochastic integral of operator-valued random functions. Let $(W_t)_{t\in[0,1]}$ be a Wiener process in X, the covariance operator of W_1 be R, and $(\mathscr{F}_t)_{t\in[0,1]}$ be a family of σ-algebras adapted to $(W_t)_{t\in[0,1]}$. Let Y be another separable Banach space, Y^* be its dual, and $L(X, Y)$ $(L(Y^*, X^*))$ be the Banach space of bounded linear operators from X to Y (from Y^* to X^*).

Definition 3.1. A function φ: $[0, 1]\times\Omega\to L(X, Y)$ is called *non-anticipating with respect to* $(\mathscr{F}_t)_{t\in[0,1]}$ if

1. for all $x\in X$ and $y^*\in Y^*$ the real-valued function $(t, \omega)\to\langle\varphi(t, \omega)x, y^*\rangle$ is measurable;

2. for all $x\in X$, $y^*\in Y^*$, $t\in[0, 1]$ the function $\omega\to\langle\varphi(t, \omega)x, y^*\rangle$ is $\mathscr{F}_t$-measurable random variable.

Definition 3.2. We say that a non-anticipating function φ *belongs to the class* $G_R(L(X, Y))$ if

$$\sigma_R(\varphi)\equiv\sup_{\|y^*\|\leqslant 1}\Big(\int_0^1\int_\Omega\langle\varphi(t, \omega)R\varphi^*(t, \omega)y^*, y^*\rangle\,dt\,dP\Big)^{1/2}<\infty,$$

where $\varphi^*(t, \omega)\in L(Y^*, X^*)$ is the dual operator to $\varphi(t, \omega)$.

$G_R(L(X, Y))$ is the linear space and σ_R is the pseudonorm in it.

Let $\varphi\in G_R(L(X, Y))$, and take any $y^*\in Y^*$. φ^*y^* maps $[0, 1]\times\Omega$ into X^* and $\varphi^*y^*\in G_R(X^*)$. Therefore we can define the stochastic integral $\int_0^1\varphi^*y^*\,dW(t)$ which will be a real random variable with variance

$$\int_0^1\int_\Omega\langle R\varphi^*y^*, \varphi^*y^*\rangle\,dt\,dP.$$

Consider the map

$$T_\varphi:\ Y^*\to L_2(\Omega, \mathscr{B}, P),\qquad T_\varphi y^*=\int_0^1\varphi^*y^*\,dW(t).$$

It is easy to see that T_φ is a linear continuous map, i.e., it is a random linear function (RLF).

Definition 3.3. Let $\varphi\in G_R(L(X, Y))$. The linear continuous map (RLF) T_φ: $Y^*\to L_2(\Omega, \mathscr{B}, P)$, defined by

$$T_\varphi y^*=\int_0^1\varphi^*y^*\,dW(t),\qquad y^*\in Y^*,$$

is called the *generalized stochastic integral* of operator-valued random function φ with respect to $(W_t)_{t\in[0,1]}$.

This implies that for any function $\varphi\in G_R(L(X, Y))$ there exists the generalized stochastic integral of φ.

Let $\varphi\in G_R(L(X, Y))$, T_φ: $Y^*\to L_2(\Omega, \mathscr{B}, P)$ be a generalized stochastic integral of φ. Define by L_φ: $Y^*\to Y^{**}$ the covariance operator of the generalized stochastic integral (RLF) (see for example [2]). It is clear that $L_\varphi = T_\varphi^* T_\varphi$.

THEOREM 3.1. *The covariance operator of the generalized stochastic integral of an operator-valued random function $\varphi \in G_R(L(X, Y))$ with respect to the Wiener process $(W_t)_{t\in[0,1]}$ has the form*

$$L_\varphi(y^*) = \int_0^1 \int_\Omega \varphi R \varphi^* y^* \, dt \, dP$$

and maps Y^* into Y (the double integral is meant in the sence of Pettis).

Proof. Let us find the value of the operator L_φ on $y^* \in Y^*$. For any $y_1^* \in Y^*$ we have

$$\langle L_\varphi y^*, y_1^* \rangle = E T_\varphi y^* T_\varphi y_1^* = E\Big(\int_0^1 \varphi^* y^* \, dW(t) \int_0^1 \varphi^* y_1^* \, dW(t)\Big).$$

It is not difficult to see that

$$\langle L_\varphi y^*, y_1^* \rangle = \int_0^1 \int_\Omega \langle \varphi R \varphi^* y^*, y_1^* \rangle \, dt \, dP \quad \text{for all } y_1^* \in Y^*.$$

Therefore the Pettis integral

$$\int_0^1 \int_\Omega \varphi R \varphi^* y^* \, dt \, dP \equiv L_\varphi y^*,$$

as an element of Y^{**}, exists for any $y^* \in Y^*$. We shall prove that $L_\varphi y^* \in Y$ ($Y \subset Y^{**}$ is understood in the sense of the natural imbedding). Let

$$Rx^* = \sum_{k=1}^\infty \langle Q_k, x^* \rangle Q_k$$

be an expansion of the covariance operator (see for example [17], p. 17). Then

$$L_\varphi y^* = \int_0^1 \int_\Omega \Big(\sum_{k=1}^\infty \langle \varphi Q_k, y^* \rangle \varphi Q_k\Big) dt \, dP.$$

For all $n \in N$ define

$$L_\varphi^{(n)} y^* \equiv \int_0^1 \int_\Omega \Big(\sum_{k=1}^n \langle \varphi Q_k, y^* \rangle \varphi Q_k\Big) dt \, dP.$$

Consider the random element φQ_k: $[0, 1] \times \Omega \to Y$, $k = 1, 2, \ldots$ Since φQ_k is a random element with the weak second order, its covariance operator maps Y^* into Y ([17], th. 7). We have for the covariance operator of φQ_k:

$$L_k y^* = \int_0^1 \int_\Omega \langle \varphi Q_k, y^* \rangle \varphi Q_k \, dt \, dP.$$

Therefore, for all $n \in N$ and $y^* \in Y^*$, $L_\varphi^{(n)} y^*$ belongs to Y. Since Y is a closed subspace of Y^{**}, it suffices to prove that the sequence $L_\varphi^{(n)} y^*$, $n = 1, 2, \ldots$, converges to $L_\varphi y^*$ in Y^{**} for all $y^* \in Y^*$. We have

$$\|L_\varphi^{(n)} y^* - L_\varphi y^*\|_{Y^{**}} = \Big\|\int_0^1 \int_\Omega \Big(\sum_{k=n+1}^{\infty} \langle \varphi Q_k, y^* \rangle \varphi Q_k \Big) dt\, dP \Big\|_{Y^{**}}$$

$$\leqslant \sup_{\|y_1^*\| \leqslant 1} \Big(\int_0^1 \int_\Omega \sum_{k=n+1}^{\infty} \langle Q_k, \varphi^* y_1^* \rangle^2 dt\, dP\Big)^{1/2} \times$$

$$\times \Big(\int_0^1 \int_\Omega \sum_{k=n+1}^{\infty} \langle \varphi^* y^*, Q_k \rangle^2 dt\, dP\Big)^{1/2}.$$

Since

$$\infty > \int_0^1 \int_\Omega \langle R\varphi^* y^*, \varphi^* y^* \rangle dt\, dP = \int_0^1 \int_\Omega \Big(\sum_{k=1}^{\infty} \langle Q_k, \varphi^* y^* \rangle^2 \Big) dt\, dP,$$

we have

$$\int_0^1 \int_\Omega \Big(\sum_{k=n+1}^{\infty} \langle Q_k, \varphi^* y^* \rangle^2 \Big) dt\, dP \to 0 \qquad \text{for } n \to \infty.$$

Further,

$$\sup_{\|y_1^*\| \leqslant 1} \Big(\int_0^1 \int_\Omega \sum_{k=n+1}^{\infty} \langle Q_k, \varphi^* y_1^* \rangle^2 dt\, dP\Big)^{1/2} \leqslant \sigma_R(\varphi) < \infty.$$

Consequently, $L_\varphi^{(n)} y^* \to L_\varphi y^*$ for $n \to \infty$. Therefore $L_\varphi y^* \in Y$. Theorem 3.1 is proved.

The generalized stochastic integral, as an RLF, induces a cylindrical measure on Y which, obviously, cannot always be extended to a countably additive measure on the Borel σ-algebra $\mathscr{B}(Y)$. In other words, T_φ is not always decomposable, i.e. there does not exist in general a random element $\xi\colon \Omega \to Y$ such that $T_\varphi y^* = \langle \xi, y^* \rangle$, $y^* \in Y^*$.

Definition 3.4. Let φ be an operator-valued function, $\varphi \in G_R(L(X, Y))$. We say that a random element $\xi\colon \Omega \to Y$ (if such an element exists) is the *stochastic integral of φ with respect to a Wiener process* $(W_t)_{t \in [0,1]}$ if $\langle \xi, y^* \rangle = T_\varphi y^*$ a.e. for all $y^* \in Y^*$, and write

$$\xi = \int_0^1 \varphi\, dW(t).$$

Thus the question of the existence of a stochastic integral is reduced to the well-known problem on extension of cylindrical measures to countably-additive measures or, equivalently, to the problem of decomposability of RLF.

In the concluding part of the paper we deal with a sufficient condition for the existence of a stochastic integral. The main point here is the use of the L. Schwartz's theorem saying that an operator between Banach spaces is p-Radonfying if and only if it is p-absolutely summing ($1 < p < \infty$) (see [15], and [9]).

Definition 3.5. *We say that a symmetric and positive operator* L: $Y^* \to Y$ *belongs to the class* $\mathscr{R}_p(Y)$ ($0 < p < \infty$) if the operator T: $H \to Y$ in the factorization $L = TT^*$ is p-absolutely summing.

It is easy to see that $\mathscr{R}_p(Y) \subset R_{p_1}(Y)$ if $p < p_1$.

THEOREM 3.2. *Let* $\varphi \in G_R(L(X, Y))$ *and let a closed subspace* G *of* $L_2(\Omega, \mathscr{B}, P)$ *be such that, for all* $y^* \in Y^*$,

$$T_\varphi y^* = \int_0^1 \varphi^* y^* \, dW(t)$$

belongs to G *and* $G \subset L_p(\Omega, \mathscr{B}, P)$, $p \geqslant 2$.

If the operator L_φ: $Y^* \to Y$,

$$L_\varphi y^* = \int_0^1 \int_\Omega \varphi R \varphi^* y^* \, dt \, dP,$$

belongs to $\mathscr{R}_p(Y)$, *then there exists a stochastic integral* $\int_0^1 \varphi \, dW(t)$ *and*

$$E \Big\| \int_0^1 \varphi \, dW(t) \Big\|^p < \infty.$$

Proof. We set $H_0 \equiv \{T_\varphi y^*: \ y^* \in Y^*\}$, $H_0 \subset L_2(\Omega, \mathscr{B}, P)$. Let H be the closure of H_0 in $L_2(\Omega, \mathscr{B}, P)$. H is a Hilbert space and $H \subset G$, therefore $H \subset L_p(\Omega, \mathscr{B}, P)$. Factorize now the operator L_φ: $Y^* \to Y$ through the Hilbert space H: $L_\varphi = T_\varphi T_\varphi^*$. Since $L_\varphi \in \mathscr{R}_p(Y)$, T_φ^*: $H \to Y$ is P-absolutely summing. Let C: $H \to L_p(\Omega, \mathscr{B}, P)$ be the natural imbedding: $Ch = h \in L_p(\Omega, \mathscr{B}, P)$. By the closed graph theorem, C is bounded. According to the aforementioned theorem of L. Schwartz, there exists a random element $\eta \in L_p(\Omega, Y)$ such that, for all $y^* \in Y^*$, $CT_\varphi y^* = \langle \eta, y^* \rangle$. Since

$$CT_\varphi y^* = \int_0^1 \varphi^* y^* \, dW(t),$$

we have, for every $y^* \in Y^*$,

$$\langle \eta, y^* \rangle = \int_0^1 \varphi^* y^* \, dW(t),$$

i.e.

$$\eta = \int_0^1 \varphi \, dW(t).$$

Theorem 3.2 is proved.

Remark. If X and Y are separable Hilbert spaces, then the condition of

Theorem 3.2 is equivalent to the condition of Yu. Daletzky [3] in his definition of stochastic integral in the Hilbert space. If X and Y are separable Banach spaces, then the condition of Theorem 3.2 in the case $p = 2$ is equivalent to the condition of H. Kuo [7]. The most interesting is the case where $p > 2$.

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