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MARTINGALE CHARACTERIZATIONS OF STOCHASTIC PROCESSES ON COMPACT GROUPS

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Abstract: By a classical result of P. Lévy, the Brownian motion $(B_t)_{t\geq 0}$ on $\mathbf{R}$ may be characterized as a continuous process on $\mathbf{R}$ such that $(B_t)_{t\geq 0}$ and $(B_t^2 - t)_{t\geq 0}$ are martingales. Generalizations of this result are usually obtained in the setting of the so-called martingale problem. This paper contains a variant of the martingale problem for stochastic processes on locally compact groups with independent stationary increments that is based on irreducible unitary representations. In particular, for Gaussian processes on compact Lie groups, analogues of the Lévy-characterization above are obtained. It turns out that for certain compact Lie groups even the continuity assumption in this characterization can be dropped.

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