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A GENERAL CONTRACTION PRINCIPLE FOR VECTOR-VALUED MARTINGALES

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Abstract: We prove a contraction principle for vector-valued martingales of type

$$\Big\| \sum_{i=1}^{n} \Delta_i x_i \Big\|_{L_p^X} \leq c_p \Big\| \sup_{1\leq i\leq n} A_i(\Delta_i) \Big\|_{L_p} \Big\| \sum_{i=1}^{n} H_i x_i \Big\|_{L_1^X} \quad (1 \leq p < \infty),$$

where X is a Banach space with elements $x_1, \ldots, x_n$, $(\Delta_i)_{i=1}^n \subset L_1(Q, \mathbf{P})$ a martingale difference sequence belonging to a certain class, $(H_i)_{i=1}^n \subset L_1(M, \nu)$ a sequence of independent and symmetric random variables exponential in a certain sense, and A_i operators mapping each Δ_i into a non-negative random variable. Moreover, special operators A_i are discussed and an application to Banach spaces of Rademacher type α $(1 < \alpha \leq 2)$ is given.

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