PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 21, Fasc. 2 (2001), pp. 441–465

MAX-SEMISTABLE HEMIGROUPS: STRUCTURE, DOMAINS OF ATTRACTION AND LIMIT THEOREMS WITH RANDOM SAMPLE SIZE

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Abstract: Let (X_n) be a sequence of independent real valued random variables. A suitable convergence condition for affine normalized maxima of (X_n) is given in the semistable setup, i.e. for increasing sampling sequences (k_n) such that $k_{n+1}/k_n \to c > 1$, which enables us to obtain a hemigroup structure in the limit. We show that such hemigroups are closely related to max-semiselfdecomposable laws and that the norming sequences of the convergence condition can be chosen such that the limiting behaviour for arbitrary sampling sequences can be fully analysed. This in turn enables us to obtain randomized limits as follows. Suppose that (T_n) is a sequence of positive integer valued random variables such that T_n/k_n or T_n/n converges in probability to some positive random variable D, where we do not assume (X_n) and (T_n) to be independent. Then weak limit theorems of randomized extremes, where the sampling sequence (k_n) is replaced by random sample sizes (T_n), are presented. The proof follows corresponding results on the central limit theorem, containing the verification of an Anscombe condition.

1991 AMS Mathematics Subject Classification: Primary 60G70; Secondary 60F05, 60E07.

Key words and phrases: Extreme values, max-semistable distributions, hemigroup, max-semiselfdecomposability, random sample size, randomized limit theorem, Anscombe condition.

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