A NOTE ON INVARIANT SETS

René L. Schilling

Abstract: A measurable set A is *invariant* with respect to a not necessarily symmetric sub-Markovian operator T on $L^p(X, m)$ if $T\mathbf{I}_A \leq \mathbf{1}_A$, and *strongly invariant* if $T\mathbf{I}_A = \mathbf{1}_A$. We show that these definitions accommodate many of the usual definitions of invariance, e.g., those used in Dirichlet form theory, ergodic theory or for stochastic processes. In finite measure spaces or if T^* is sub-Markovian and recurrent, the notions of invariance and strong invariance coincide. We also show that for certain analytic semigroups of sub-Markovian operators, (strongly) invariant sets are already determined by a single operator, T_1.

2000 AMS Mathematics Subject Classification: 31C15,47D07, 60J35, 60J45.

Key words and phrases: Invariant set, sub-Markov operator, recurrence, transience, fractional power, analytic semigroup.

THE FULL TEXT IS AVAILABLE HERE