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ON APPROXIMATIONS OF RISK PROCESS WITH RENEWAL ARRIVALS IN α-STABLE DOMAIN

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Abstract. In this paper we approximate risk process by an α-stable Lévy motion ($1 < \alpha \leqslant 2$). We consider two conditions imposed on the value of the premium rate. The first one assumes that the premiums exceed only slightly the expected claims (heavy traffic) and the second one assumes that the premiums are much larger than the average claims (light traffic). We consider the distribution of claim sizes belonging to the domain of attraction of an α-stable law and the process counting claims is a renewal process constructed from random variables belonging to the domain of attraction of an α'-stable law. Comparing α and α' we obtain three different asymptotic risk processes. In the classical model we get a Brownian diffusion approximation which fits first two moments. If $\alpha' > \alpha$, we get Mittag-Leffler distribution for the infinite time ruin probability, and if $\alpha' < \alpha$, we obtain exponential distribution for the infinite time ruin probability.

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1. INTRODUCTION

Collective risk theory is concerned with random fluctuations of the total net assets – the capital of an insurance company. Consider a company which only writes ordinary insurance policies such as accident, disability, health and whole life. The policyholders pay premiums regularly and at certain random times make claims to the company. A policyholder's premium, the gross risk premium, is a positive amount composed of two components. The net risk premium is the component calculated to cover the payments of claims on the average, while the security risk premium, or safety loading, is the component which protects the company from large deviations of claims from the average and also allows an accumulation of capital. So the risk process has the Cramér–Lundberg form $R(t) = u + pt - \sum_{k=1}^{N_t} Y_k$, where $u > 0$ is the initial risk reserve of the company and the policyholders pay a gross risk premium of $p > 0$ per a unit time. The successive claims $\{Y_k\}$ are assumed to form a sequence of i.i.d. random variables with mean $EY_1 = \mu$ and claims occur at jumps of a renewal counting process N_t, $t \geqslant 0$.

The *ruin time* T is defined as the first time the company has a negative risk reserve and one of the key problems of collective risk theory concerns calculating the *ultimate ruin probability* $\Psi = P\big(T < \infty \mid R(0) = u\big)$, i.e. the probability that the risk process ever becomes negative. On the other hand, an insurance company will typically be interested in the probability that ruin occurs before time t, i.e. $\Psi(t) = P\big(T \leqslant t \mid R(0) = u\big)$. Many of the results are in the form of complicated analytic expressions. For a comprehensive treatment of the theory the reader should consult Asmussen [1], Embrechts et al. [5] or Rolski et al. [15].

Diffusion approximation of random walks via Donsker's theorem is a classical topic of probability theory. The first application in risk theory is the paper by Iglehart [10], and two further standard references in the area are Grandell [8] and [9]. For claims with infinite variance the paper by Furrer et al. [7] suggested an approximation by an α-stable Lévy motion. The idea is to let the number of claims grow in a unit time interval and to make the claim sizes smaller in such a way that the risk process converges weakly to an α-stable Lévy motion with a linear drift.

In this paper we consider two settings of the premium with respect to the average claims, that is heavy and light traffic. The terms 'light' and 'heavy traffic' come from queueing theory and have an obvious interpretation. *Light traffic* means that the premium is much larger than the average amount of claims per unit time and *heavy traffic* describes the case when the premium rate exceeds only slightly the average amount of claims per unit time. Thus we approximate a risk process with renewal arrivals by an α-stable Lévy motion in these two settings. The stability parameter α depends on the tails of the arrival and claim distributions and the scale parameter depends on the expected values of claims and arrivals. This work extends the results of Furrer et al. [7]. The classical risk process is approximated by a Brownian diffusion which fits first two moments.

For a similar treatment in the theory of queueing systems see Szczotka and Woyczyński [19].

2. WEAK CONVERGENCE OF STOCHASTIC PROCESSES

Let us consider the claim surplus process

$$S^{(p)}(t) = \sum_{k=1}^{N_t} Y_k - pt,$$

where $\{Y_k\}$ are i.i.d. random variables, $EY_1 = \mu$, N_t is a renewal process constructed from the sequence of i.i.d. random variables $\{T_k\}$ with $ET_1 = 1/\lambda > 0$. Moreover, we assume that $p - \lambda\mu > 0$ and, in general, the process N_t which counts claims does not have to be independent of the sequence of the claim sizes $\{Y_k\}$ as assumed in many risk models. Then

$$R(t) = u - S^{(p)}(t)$$

is a risk process. If N_t is Poisson process independent of the sequence of the claim sizes $\{Y_k\}$, we get the classical risk model.

Assume that the sequence $\{Y_k\}$ satisfies

$$\frac{1}{\phi(n)} \sum_{k=1}^{n} (Y_k - \mu) \Rightarrow Z_\alpha(1) \tag{1}$$

when $n \to \infty$, where $EY_1 = \mu$ and $Z_\alpha(1)$ is an α-stable random variable with $1 < \alpha \leqslant 2$, scale parameter σ (for $\alpha = 2$ we put $\sigma^2 = \mathbf{Var}\, Y_1$), skewness parameter β and $\phi(n) = n^{1/\alpha} L(n)$, where $L(n)$ is a slowly varying function at infinity. Then

$$\frac{1}{\phi(c)} \sum_{k=1}^{[ct]} (Y_k - \mu) \Rightarrow Z_\alpha(t) \tag{2}$$

when $c \to \infty$ in the Skorokhod J_1 topology (see Prokhorov [14]). Moreover, we assume that the i.i.d. random variables $\{T_k\}$ fulfill

$$\frac{1}{\phi'(n)} \sum_{k=1}^{n} (T_k - 1/\lambda) \Rightarrow Z'_{\alpha'}(1) \tag{3}$$

when $n \to \infty$, where $ET_1 = 1/\lambda > 0$ and $Z'_{\alpha'}(1)$ is an α'-stable random variable with $1 < \alpha' \leqslant 2$, scale parameter σ' (for $\alpha' = 2$ we put $\sigma'^2 = \mathbf{Var}\, T_1$), skewness parameter β' and $\phi'(n) = n^{1/\alpha'} L'(n)$, where $L'(n)$ is a slowly varying function at infinity. Then

$$\frac{N_{ct} - \lambda ct}{\phi'(c)} \Rightarrow -\lambda^{1+1/\alpha'} Z'_{\alpha'}(t) \tag{4}$$

when $c \to \infty$ in the Skorokhod M_1 topology (see Bingham [4] or Michna [13]).

PROPOSITION 1. *Assume that the sequences $\{Y_k\}$ and $\{T_k\}$ satisfy* (1) *and* (3) *and that $\alpha < \alpha'$. Then*

$$\frac{1}{\phi(c)} \Big(\sum_{k=1}^{N_{ct}} Y_k - \lambda\mu ct \Big) \Rightarrow Z_\alpha(\lambda t)$$

when $c \to \infty$ in the Skorokhod J_1 topology.

Proof. We have

$$\frac{1}{\phi(c)}\Big(\sum_{k=1}^{N_{ct}} Y_k - \lambda\mu ct\Big) = \frac{1}{\phi(c)}\sum_{k=1}^{N_{ct}} (Y_k-\mu)+\mu\frac{N_{ct}-\lambda ct}{\phi(c)} \Rightarrow Z_\alpha(\lambda t)$$

because

$$\frac{1}{\phi(c)}\sum_{k=1}^{N_{ct}} (Y_k-\mu) \Rightarrow Z_\alpha(\lambda t)$$

(note that $N_{ct}/c \Rightarrow \lambda t$ as $c\to\infty$ and use Theorem 3.1 of Whitt [22]) and moreover

$$\frac{N_{ct}-\lambda ct}{\phi'(c)} \Rightarrow -\lambda^{1+1/\alpha'} Z'_{\alpha'}(t)$$

in the Skorokhod M_1 topology, and $\phi'(c)/\phi(c)\to 0$ when $c\to\infty$, which implies

$$\frac{N_{ct}-\lambda ct}{\phi(c)} \Rightarrow 0. \ \blacksquare$$

Similarly we can show the following proposition.

PROPOSITION 2. *Assume that the sequences* $\{Y_k\}$ *and* $\{T_k\}$ *satisfy* (1) *and* (3) *and that* $\alpha' < \alpha$. *Then*

$$\frac{1}{\phi'(c)}\Big(\sum_{k=1}^{N_{ct}} Y_k - \lambda\mu ct\Big) \Rightarrow -\mu\lambda^{1+1/\alpha'} Z'_{\alpha'}(t)$$

when $c\to\infty$ *in the Skorokhod* M_1 *topology.*

In the case $\alpha=\alpha'$ the problem is more complicated but it can be solved.

PROPOSITION 3. *Assume that the sequences* $\{Y_k\}$ *and* $\{T_k\}$ *satisfy* (1) *and* (3) *with* $\phi(n)\equiv\phi'(n)$ *and are independent. Then*

$$\frac{1}{\phi(c)}\Big(\sum_{k=1}^{N_{ct}} Y_k - \lambda\mu ct\Big) \Rightarrow \lambda^{1/\alpha} Z_\alpha(t) - \mu\lambda^{1+1/\alpha} Z'_\alpha(t)$$

when $c\to\infty$ *in the Skorokhod* M_1 *topology.*

Proof. The assumption $\phi(n)\equiv\phi'(n)$ implies that $\alpha=\alpha'$, and thus

$$\frac{1}{\phi(c)}\Big(\sum_{k=1}^{N_{ct}} Y_k - \lambda\mu ct\Big) = \frac{1}{\phi(c)}\sum_{k=1}^{N_{ct}} (Y_k-\mu)+\mu\frac{N_{ct}-\lambda ct}{\phi(c)}$$
$$\Rightarrow \lambda^{1/\alpha} Z_\alpha(t) - \mu\lambda^{1+1/\alpha} Z'_\alpha(t)$$

because

$$\frac{1}{\phi(c)}\sum_{k=1}^{N_{ct}} (Y_k-\mu) \Rightarrow Z_\alpha(\lambda t) \quad\text{and}\quad \frac{N_{ct}-\lambda ct}{\phi(c)} \Rightarrow -\lambda^{1+1/\alpha} Z'_\alpha(t)$$

when $c\to\infty$ in the Skorokhod M_1 topology. Thus, using Theorem 5.1 of Whitt [22], we complete the proof. $\blacksquare$

3. WEAK APPROXIMATION OF CLAIM SURPLUS PROCESS

In this section we will approximate the claim surplus process by an α-stable Lévy motion in two settings: heavy and light traffic. Now we are in a position to rewrite Propositions 1, 2 and 3 in terms of the claim surplus process. For the clear presentation let us introduce the following notation:

$$c_\alpha(p) = (p-\lambda\mu)^{-\alpha/(\alpha-1)} \tag{5}$$

and similarly

$$c_{\alpha'}(p) = (p-\lambda\mu)^{-\alpha'/(\alpha'-1)}. \tag{6}$$

We first consider heavy traffic approximation, that is, the premiums exceed only slightly the expected claims on the average.

THEOREM 4. *Assume that the sequences $\{Y_k\}$ and $\{T_k\}$ belong to the domain of normal attraction of a stable law, that is, $\phi(n) = n^{1/\alpha}$ and $\phi'(n) = n^{1/\alpha'}$. Let the functions $c_\alpha(p)$ and $c_{\alpha'}(p)$ be defined in* (5) *and* (6), *respectively. Then, as $p \downarrow \lambda\mu$,*

$$\big(c_\alpha(p)\big)^{-1/\alpha} S^{(p)}\big(tc_\alpha(p)\big) \Rightarrow \lambda^{1/\alpha} Z_\alpha(t) - t$$

in the Skorokhod J_1 topology when $\alpha < \alpha'$, and

$$\big(c_{\alpha'}(p)\big)^{-1/\alpha'} S^{(p)}\big(tc_{\alpha'}(p)\big) \Rightarrow -\mu\lambda^{1+1/\alpha'} Z'_{\alpha'}(t) - t$$

in the Skorokhod M_1 topology when $\alpha' < \alpha$, and

$$\big(c_\alpha(p)\big)^{-1/\alpha} S^{(p)}\big(tc_\alpha(p)\big) \Rightarrow \lambda^{1/\alpha} Z_\alpha(t) - \mu\lambda^{1+1/\alpha} Z'_\alpha(t) - t$$

in the Skorokhod M_1 topology when $\alpha' = \alpha$ and the sequences $\{Y_k\}$ and $\{T_k\}$ are independent.

Proof. Using the identity

$$\sum_{k=1}^{N_{ct}} Y_k - \lambda\mu ct = S^{(p)}(ct) + ct(p-\lambda\mu)$$

and substituting $c = c_\alpha(p) = (p-\lambda\mu)^{-\alpha/(\alpha-1)}$ in Proposition 1 for $\alpha < \alpha'$, we get the desired convergence. Similarly, using Propositions 2 and 3 we show the convergence for $\alpha' < \alpha$ and $\alpha = \alpha'$. ∎

In the light traffic approximation we assume that the premiums are much larger than the expected claims on the average.

THEOREM 5. *Assume that the sequences $\{Y_k\}$ and $\{T_k\}$ belong to the domain of normal attraction of a stable law, that is, $\phi(n) = n^{1/\alpha}$ and $\phi'(n) = n^{1/\alpha'}$. Let the functions $c_\alpha(p)$ and $c_{\alpha'}(p)$ be defined in* (5) *and* (6), *respectively. Then, as $p \to \infty$,*

$$\big(c_\alpha(p)\big)^{1/\alpha} S^{(p)}\big(t/c_\alpha(p)\big) + (p-\lambda\mu)^2 t \Rightarrow \lambda^{1/\alpha} Z_\alpha(t)$$

in the Skorokhod J_1 topology when $\alpha < \alpha'$, and

$$\big(c_{\alpha'}(p)\big)^{1/\alpha'} S^{(p)}\big(t/c_{\alpha'}(p)\big) + (p-\lambda\mu)^2 t \Rightarrow -\mu\lambda^{1+1/\alpha'} Z'_{\alpha'}(t)$$

in the Skorokhod M_1 topology when $\alpha' < \alpha$, and

$$(c_\alpha(p))^{1/\alpha} S^{(p)}(t/c_\alpha(p)) + (p-\lambda\mu)^2 t \Rightarrow \lambda^{1/\alpha} Z_\alpha(t) - \mu\lambda^{1+1/\alpha} Z'_\alpha(t)$$

in the Skorokhod M_1 topology when $\alpha' = \alpha$ and the sequences $\{Y_k\}$ and $\{T_k\}$ are independent.

Proof. As before, using the identity

$$\sum_{k=1}^{N_{ct}} Y_k - \lambda\mu ct = S^{(p)}(ct) + ct(p-\lambda\mu)$$

and substituting $c = 1/c_\alpha(p) = (p-\lambda\mu)^{\alpha/(\alpha-1)}$ in Proposition 1 for $\alpha < \alpha'$, we get the desired convergence. Similarly, using Propositions 2 and 3 we show the convergence for $\alpha' < \alpha$ and $\alpha = \alpha'$. ■

4. EXAMPLES

Now we want to take advantage of the above results, and therefore we present the crucial theorems. For the finite time ruin probability we need the following result.

THEOREM 6. *Under the assumptions of Theorem 4, as $p \downarrow \lambda\mu$,*

$$P\left(\sup_{s\leqslant t} (c_\alpha(p))^{-1/\alpha} S^{(p)}(sc_\alpha(p)) > u\right) \to P\left(\sup_{s\leqslant t} (X(s)-s) > u\right)$$

for every $t > 0$ when $\alpha \leqslant \alpha'$ and

$$X(s) = \begin{cases} \lambda^{1/\alpha} Z_\alpha(s) & \text{if } \alpha < \alpha', \\ \lambda^{1/\alpha} Z_\alpha(s) - \mu\lambda^{1+1/\alpha} Z'_\alpha(s) & \text{if } \alpha = \alpha'. \end{cases} \tag{7}$$

Similarly, as $p \downarrow \lambda\mu$,

$$P\left(\sup_{s\leqslant t} (c_{\alpha'}(p))^{-1/\alpha'} S^{(p)}(sc_{\alpha'}(p)) > u\right) \to P\left(\sup_{s\leqslant t} (X(s)-s) > u\right)$$

for every $t > 0$ when $\alpha' < \alpha$ and

$$X(s) = -\mu\lambda^{1+1/\alpha'} Z'_{\alpha'}(s). \tag{8}$$

Proof. In the Gaussian case, that is, $\alpha = \alpha' = 2$, the assertion follows from Proposition 5 of Michna [12], and in other cases from Theorem 2 of Furrer et al. [7]. ■

The convergence of infinite time ruin probabilities is much more cumbersome because the functional $\sup_{s>0} x(s)$ is not continuous in Skorokhod topologies.

THEOREM 7. *Under the assumptions of Theorem 4, as $p \downarrow \lambda\mu$,*

$$P\left(\sup_{s>0} (c_\alpha(p))^{-1/\alpha} S^{(p)}(sc_\alpha(p)) > u\right) \to P\left(\sup_{s>0} (X(s)-s) > u\right)$$

when $\alpha \leqslant \alpha'$ *and the process* X *is defined in* (7). *Similarly, as* $p \downarrow \lambda\mu$,

$$P\left(\sup_{s>0} (c_{\alpha'}(p))^{-1/\alpha'} S^{(p)}(sc_{\alpha'}(p)) > u\right) \to P\left(\sup_{s>0} (X(s) - s) > u\right)$$

when $\alpha' < \alpha$ *and the process* X *is defined in* (8).

Proof. In the Gaussian case, that is, $\alpha = \alpha' = 2$ the assertion follows from Theorem 7.1 in Asmussen [2] and in the α-stable case, that is, $1 < \alpha < 2$, from Theorem 3 in Szczotka and Woyczyński [18] (using duality of ruin probability and distribution of stationary waiting time for $G/G/1$ queues, see e.g. Asmussen [1]). ∎

Thus let us consider heavy traffic approximation of a claim surplus process and approximate ruin probability. For $\alpha \leqslant \alpha'$ and $p \downarrow \lambda\mu$ we obtain

$$\begin{aligned} \Psi(t) &= P\left(\sup_{s \leqslant t} S^{(p)}(s) > u\right) = P\left(\sup_{s \leqslant t/c_\alpha(p)} S^{(p)}(sc_\alpha(p)) > u\right) \\ &= P\left(\sup_{s \leqslant t/c_\alpha(p)} (c_\alpha(p))^{-1/\alpha} S^{(p)}(sc_\alpha(p)) > (c_\alpha(p))^{-1/\alpha} u\right) \\ &\approx P\left(\sup_{s \leqslant t/c_\alpha(p)} (X(s) - s) > (c_\alpha(p))^{-1/\alpha} u\right) = P\left(\sup_{s \leqslant t} (X(s) - (p - \lambda\mu)s) > u\right), \end{aligned}$$

where the process X is defined in (7), in the second last equality we have used Theorem 6 and in the last equality the self-similarity property of the process X. Similarly we proceed for $\alpha' < \alpha$ and for infinite time ruin probabilities using Theorem 7. Thus the infinite time ruin probability for $p \downarrow \lambda\mu$ can be approximated in the following way:

$$\Psi = P\left(\sup_{s>0} S^{(p)}(s) > u\right) \approx P\left(\sup_{s>0} (X(s) - (p - \lambda\mu)s) > u\right)$$

and the finite time ruin probability as

$$\Psi(t) = P\left(\sup_{s \leqslant t} S^{(p)}(s) > u\right) \approx P\left(\sup_{s \leqslant t} (X(s) - (p - \lambda\mu)s) > u\right),$$

where the process X is defined in (7) and (8).

Now let us focus on the case $\alpha' = \alpha = 2$ which includes the classical risk model. Then the infinite time ruin probability can be approximated in the following way:

$$\Psi \approx P\left(\sup_{s>0} \left(\sqrt{\lambda\sigma^2 + \mu^2 \lambda^3 \sigma'^2}\, B(s) - (p - \lambda\mu)s\right) > u\right) = \exp\left(-\frac{2u(p - \lambda\mu)}{\lambda\sigma^2 + \mu^2 \lambda^3 \sigma'^2}\right),$$

where $B(s)$ is the standard Brownian motion, $\sigma^2 = \mathbf{Var}\, Y_1$ and $\sigma'^2 = \mathbf{Var}\, T_1$. For the finite time horizon we get

$$\Psi(t) = P\big(\sup_{s \leqslant t} S^{(p)}(s) > u\big) \approx 1 - \Phi\left\{\frac{u + (p - \lambda\mu)t}{\sqrt{(\lambda\sigma^2 + \mu^2\lambda^3\sigma'^2)t}}\right\}$$

$$+ \exp\left\{-\frac{2u(p-\lambda\mu)}{\lambda\sigma^2 + \mu^2\lambda^3\sigma'^2}\right\}\left[1 - \Phi\left\{\frac{u - (p-\lambda\mu)t}{\sqrt{(\lambda\sigma^2 + \mu^2\lambda^3\sigma'^2)t}}\right\}\right],$$

where $\Phi(x)$ is the standard normal distribution. In the classical case, when N_t is a Poisson process with intensity λ ($\mathbf{Var}\, T_1 = \sigma'^2 = 1/\lambda^2$), we obtain

$$\Psi \approx P\big(\sup_{s>0} (\sqrt{\lambda(\sigma^2+\mu^2)}\, B(s) - (p-\lambda\mu)s) > u\big) = \exp\left(-\frac{2u(p-\lambda\mu)}{\lambda(\sigma^2+\mu^2)}\right)$$

and

$$\Psi(t) = P\big(\sup_{s \leqslant t} S^{(p)}(s) > u\big)$$

$$\approx 1 - \Phi\left\{\frac{u+(p-\lambda\mu)t}{\sqrt{(\lambda(\sigma^2+\mu^2))t}}\right\} + \exp\left\{-\frac{2u(p-\lambda\mu)}{\lambda(\sigma^2+\mu^2)}\right\}\left[1 - \Phi\left\{\frac{u-(p-\lambda\mu)t}{\sqrt{(\lambda(\sigma^2+\mu^2))t}}\right\}\right].$$

Interesting results can be obtained in the case $\alpha' < \alpha$. We do not assume that the interarrivals $\{T_k\}$ are positive random variables, only their expectation has to be positive. But if we assume that $\{T_k\}$ are positive random variables, then the limit process has the skewness parameter equal to -1. Thus, as $p \downarrow \lambda\mu$, the infinite time ruin probability has the following form:

$$\Psi \approx P\big(\sup_{s>0} (-\mu\lambda^{1+1/\alpha'} Z'_{\alpha'}(s) - (p-\lambda\mu)s) > u\big) = \exp(-a^{1/(\alpha'-1)} u),$$

where $a = (p-\lambda\mu)(\mu\sigma')^{-\alpha'}\lambda^{-\alpha'-1}\cos\{\pi(\alpha'-2)/2\}$.

In the case $\alpha < \alpha'$, if the claim sizes $\{Y_k\}$ are positive random variables, as $p \downarrow \lambda\mu$, we obtain the following approximation for the infinite time ruin probability:

$$\Psi \approx P\big(\sup_{s>0} \lambda^{1/\alpha} Z_\alpha(s) - (p-\lambda\mu)s > u\big) = \sum_{n=0}^{\infty} \frac{(-a)^n}{\Gamma\{1+(\alpha-1)n\}} u^{(\alpha-1)n},$$

where $a = (p-\lambda\mu)\sigma^{-\alpha}\lambda^{-1}\cos\{\pi(\alpha-2)/2\}$ (see Furrer [6] or Szczotka and Woyczyński [18]).

In the case $\alpha = \alpha'$ and $1 < \alpha < 2$ it is possible to obtain in the limit symmetric α-stable Lévy motion, and the exact form of the Laplace–Stieltjes transform of the infinite time ruin distribution ($G(u) = 1 - \Psi(u)$) is given in Szczotka and Woyczyński [18].

REFERENCES

[1] S. Asmussen, *Ruin Probabilities*, World Scientific, Singapore 2000.

[2] S. Asmussen, *Applied Probability and Queues*, Springer, New York 2003.

[3] P. Billingsley, *Convergence of Probability Measures*, Wiley, New York 1968.

[4] N. H. Bingham, *Maxima of sums of random variables and suprema of stable processes*, Z. Wahrsch. verw. Gebiete 26 (1973), pp. 273–293.

[5] P. Embrechts, C. Klüppelberg and T. Mikosch, *Modeling Extremal Events. With a View towards Insurance and Finance*, Springer, Berlin 1997.

[6] H. Furrer, *Risk processes perturbed by α-stable Lévy motion*, Scand. Actuar. J. 10 (1998), pp. 23–35.

[7] H. Furrer, Z. Michna and A. Weron, *Stable Lévy motion approximation in collective risk theory*, Insurance: Math. Econom. 20 (1997), pp. 97–114.

[8] J. Grandell, *A class of approximations of ruin probabilities*, Scand. Actuar. J., Suppl. (1977), pp. 37–52.

[9] J. Grandell, *A remark on 'A class of approximations of ruin probabilities'*, Scand. Actuar. J. (1978), pp. 77–78.

[10] D. L. Iglehart, *Diffusion approximations in collective risk theory*, J. Appl. Probab. 6 (1969), pp. 285–292.

[11] A. Janicki and A. Weron, *Simulation and Chaotic Behavior of α-Stable Stochastic Processes*, Marcel Dekker, New York 1994.

[12] Z. Michna, *Self-similar processes in collective risk theory*, J. Appl. Math. Stochastic Anal. 11 (1998), pp. 429–448.

[13] Z. Michna, *Alpha-stable limits for multiple channel queues in heavy traffic*, Applicationes Mathematicae 30 (2003), pp. 55–68.

[14] Y. V. Prokhorov, *Convergence of random processes and limit theorems in probability theory*, Theory Probab. Appl. 1 (1956), pp. 157–214.

[15] T. Rolski, H. Schmidli, V. Schmidt and J. Teugels, *Stochastic Processes for Insurance and Finance*, Wiley, New York 1999.

[16] G. Samorodnitsky and M. Taqqu, *Non-Gaussian Stable Processes: Stochastic Models with Infinite Variance*, Chapman and Hall, London 1994.

[17] H. Schmidli, *Diffusion approximations for a risk process with the possibility of borrowing and investment*, Stochastic Models 10 (1994), pp. 365–388.

[18] W. Szczotka and W. A. Woyczyński, *Distributions of suprema of Lévy processes via the heavy traffic invariance principle*, Probab. Math. Statist. 23 (2003), pp. 251–272.

[19] W. Szczotka and W. A. Woyczyński, *Heavy-tailed dependent queues in heavy traffic*, Probab. Math. Statist. 24 (2004), pp. 67–96.

[20] A. Weron, *Stable processes and measures: A survey*, in: *Probability Theory on Vector Spaces III*, D. Szynal and A. Weron (Eds.), Lecture Notes in Math. 1080, Springer, 1984, pp. 306–364.

[21] R. Weron, *Lévy-stable distributions revisited: tail index > 2 does not exclude the Lévy-stable regime*, Internat. J. Modern Phys. C 12 (2001), pp. 200–223.

[22] W. Whitt, *Some useful functions for functional limit theorems*, Math. Oper. Res. 5 (1980), pp. 67–81.

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