PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 25, Fasc. 2 (2005), pp. 267–278

THE FUNCTIONAL EQUATION AND STRICTLY SUBSTABLE RANDOM VECTORS

Marta Borowiecka-Olszewska

Abstract: A random vector X is β-substable, $\beta \in (0, 2]$, if there exist a symmetric β-stable random vector Y and a random variable $\Theta \geq 0$ independent of Y such that $X \stackrel{d}{=} Y\Theta^{1/\beta}$. In this paper we investigate strictly β-substable random vectors which are generated from a strictly β-stable random vector Y. We study some of their properties. We obtain the theorem that every strictly β-stable random vector X with $\Theta \sim S_{\alpha/\beta}(\sigma, 1, 0)$ is also strictly α-stable, $\alpha < \beta$ (for the case of random variable X see, e.g., [1], [6]). The opposite theorem is also satisfied, but we obtain something more. We obtain some functional equation and we show that if a strictly β-substable random vector X is α-stable, then it has to be strictly α-stable and the mixing random variable Θ has to have a distribution $S_{\alpha/\beta}(\sigma, 1, 0)$. This is the main result of the paper.

2000 AMS Mathematics Subject Classification: 60A10, 60E07, 60E10.

Key words and phrases: Stable, strictly stable, substable random vectors, spectral measure, characteristic functions.