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ON ADMISSIBLE QUADRATIC ESTIMATION IN A SPECIAL LINEAR MODEL

BY

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Abstract. A random linear model for spatially located sensors measured intensity of a source of signals in discrete instants of time is considered. A characterization of admissible quadratic estimators of the mean squared error of a linear estimator of the expectation of the intensity is given.

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1. INTRODUCTION AND NOTATION

Let $Y_1, \ldots, Y_n$ be a simple sample from the normal distribution

$$N(K\mu,\, \sigma^2 KK' + \sigma_e^2 I_n),$$

where $\mu \in \mathscr{R}$, $\sigma^2 \geqslant 0$ and $\sigma_e^2 > 0$ are unknown parameters, $K \in \mathscr{R}^J$ is a known vector, and I_n is the $n \times n$ identity matrix. Gnot et al. (2001) considered this model for spatially located sensors measured intensity of a source of signals in discrete instants of time. When $K = \mathbf{1}_J$, where $\mathbf{1}_J$ denotes the J-vector of ones, we obtain the balanced one-way classification random model. Note that $Y = (Y_1', \ldots, Y_n')'$ has the following normal distribution:

$$Y \sim N(\mu \mathbf{1}_n \otimes K,\, \sigma^2 I_n \otimes KK' + \sigma_e^2 I_n \otimes I_J), \tag{1}$$

where the symbol $\otimes$ denotes the Kronecker product. We are interested in estimation of the mean squared error of a linear estimator $l'Y$ of μ, where $l \in \mathscr{R}^{nJ}$, determined as

$$\mathrm{MSE}(l'Y) = \sigma^2 l'(I_n \otimes KK')l + \sigma_e^2 l'l + \mu^2 (l'\mathbf{1}_{nJ} - 1)^2.$$

This function is a linear combination of μ^2, σ^2 and σ_e^2. The natural class of estimators of such a function is the class of quadratic forms with respect to Y, that is

$$\{Y'AY\colon\ A\in\mathcal{S}_{nJ}\},$$

where $\mathcal{S}_{nJ}$ denotes the space of $nJ\times nJ$ symmetric matrices. To compare quadratic estimators we use again the mean squared error.

The problem of quadratic estimation of the mean squared error of a linear estimator within the Gauss–Markov model was considered for example by Stępniak (1998), Bojarski and Zmyślony (1998), Gnot and Grządziel (2002).

To characterize the class of admissible quadratic estimators we reduce first this problem to linear admissible estimation in a special linear model using a result of Zmyślony (1976). Next we use the theory developed by LaMotte (1982) to describe admissible estimators.

Under the additional assumption that $K=\mathbf{1}_J$ Zmyślony (1976), generalizing the results of Harville (1969), has shown that if $Y'DY$ is an admissible estimator, then D belongs to the quadratic subspace $\mathcal{Q}$ (Jordan algebra) spanned by

$$\mathbf{1}_n\mathbf{1}_n'\otimes KK',\ I_n\otimes KK',\ I_n\otimes I_J, \tag{2}$$

that is, D belongs to the smallest linear space $\mathcal{Q}$ containing $n\times n$ symmetric matrices such that $A^2\in\mathcal{Q}$ for every $A\in\mathcal{Q}$. This result implies that a quadratic admissible estimator is a linear combination of quadratic forms with respect to Y defined by matrices forming a base of $\mathcal{Q}$.

Under our assumptions the quadratic subspace $\mathcal{Q}$ is the linear space generated by (2). As a base of $\mathcal{Q}$ we choose

$$B_1=\frac{1}{nK'K}\mathbf{1}_n\mathbf{1}_n'\otimes KK',$$

$$B_2=\frac{1}{K'K}I_n\otimes KK'-B_1,$$

$$B_3=I_n\otimes I_J-B_1-B_2.$$

Note that

$$B_iB_j=\begin{cases}B_i & \text{for } i=j=1,2,3,\\ 0 & \text{for } i\neq j.\end{cases}$$

The result of Zmyślony (1976) implies that each admissible quadratic estimator is a linear combination of $Z=(Y'B_1Y,\ Y'B_2Y,\ Y'B_3Y)'$. It is easy to see that

$$E_\theta(Z)=TH'\theta, \tag{3}$$

where

$$T=\begin{bmatrix}t_1&0&0\\0&t_2&0\\0&0&t_3\end{bmatrix},\qquad H=\begin{bmatrix}b_1&0&0\\b_2&b_2&0\\1&1&1\end{bmatrix},\qquad \theta=\begin{bmatrix}\mu^2\\ \sigma^2\\ \sigma_e^2\end{bmatrix},$$

$t_i = \operatorname{tr}(B_i)$, $i = 1, 2, 3$, $b_1 = nK'K$, $b_2 = K'K$. Moreover, the covariance matrix of Z has the following form:

$$\text{(4)} \quad \operatorname{cov}_\theta(Z) = 2T \begin{bmatrix} (\sigma^2 b_2+\sigma_e^2)^2+2\mu^2 b_1(\sigma^2 b_2+\sigma_e^2) & 0 & 0 \\ 0 & (\sigma^2 b_2+\sigma_e^2)^2 & 0 \\ 0 & 0 & (\sigma_e^2)^2 \end{bmatrix}.$$

Of course, the vector θ is a linear combination of $E_\theta(Z)$, that is, $\theta = C' E_\theta(Z)$, where $C = T^{-1} H^{-1}$. Consequently, the mean squared error of a linear estimator $L'Z$ of θ, where L is a 3×3 matrix, can be written as

$$\text{(5)} \qquad \operatorname{MSE}(L'Z;\, \theta) = \operatorname{tr}[L' \operatorname{cov}_\theta(Z) L + (L-C)' E_\theta(Z) E_\theta(Z')(L-C)].$$

This risk depends on the distribution of Z through $\operatorname{cov}_\theta(Z)$ and $E_\theta(Z') E_\theta(Z)$ only, so we take $(\operatorname{cov}_\theta(Z), E_\theta(Z) E_\theta(Z'))$ as a new parameter. Following LaMotte (1982) we denote the new parameter space by $\mathscr{T}$. We extend the risk function (5) from $\mathscr{T}$ to $\mathscr{W} = \operatorname{span}\{\mathscr{T}\}$ for each $W = (W_1, W_2)' \in \mathscr{W}$ by

$$\text{(6)} \qquad \varrho(L'Z;\, W) = \operatorname{tr}[L' W_1 L + (L-C)' W_2 (L-C)].$$

To begin with, we present LaMotte's theorem in a form most suitable for our considerations.

Let $\mathscr{L}$ be an affine subspace of 3×3 matrices having the following form:

$$\mathscr{L} = \{L_0 + \Pi X\colon\ X \in \mathscr{M}_{3\times 3}\},$$

where $L_0,\ \Pi \in \mathscr{M}_{3\times 3}$, while $\mathscr{M}_{3\times 3}$ is the space of all 3×3 matrices. An estimator $L'Z$ with $L \in \mathscr{L}$ is called *best* among $\mathscr{L}$ at a point $W \in \mathscr{W}$ if $\varrho(L'Z;\, W) \leqslant \varrho(M'Z;\, W)$ for all $M \in \mathscr{L}$. Let $\mathscr{B}(W \mid \mathscr{L})$ denote the subset of all $L \in \mathscr{L}$ such that $L'Z$ is locally best among $\mathscr{L}$ at W. Notice that $\mathscr{B}(W \mid \mathscr{L})$ is not empty iff the matrix $\Pi'(W_1+W_2)\Pi$ is n.n.d. and

$$\text{(7)} \qquad \mathscr{R}\big(\Pi'(W_1+W_2)L_0 - \Pi' W_2 C\big) \subset \mathscr{R}\big(\Pi'(W_1+W_2)\Pi\big).$$

For a matrix M the symbol $\mathscr{R}(M)$ denotes the space generated by columns of M. If $\mathscr{B}(W \mid \mathscr{L})$ is not empty, then an estimator $L'Y$ is locally best at W among $\mathscr{L}$ iff

$$\Pi'(W_1+W_2)L = \Pi' W_2 C.$$

The last equation has the unique solution iff $\mathscr{R}\big(\Pi'(W_1+W_2)\Pi\big) = \mathscr{R}(\Pi')$.

Following LaMotte (1982), an element W in $\mathscr{W}$ is said to be a *trivial point* for $\mathscr{L}$ if $\mathscr{B}(W \mid \mathscr{L}) = \mathscr{L}$. The set of trivial points for $\mathscr{L}$ will be denoted by $\mathscr{S}(\mathscr{L})$. Obviously,

$$\mathscr{S}(\mathscr{L}) = \{W \in \mathscr{W}\colon\ \Pi'(W_1+W_2)\Pi = 0,\ \Pi'(W_1+W_2)L_0 - \Pi' W_2 C = 0\}.$$

Since $\Pi'(W_1+W_2)\Pi$ is n.n.d. for each W in the closed convex cone containing $\mathscr{T}+\mathscr{S}$, to be denoted by $[\mathscr{T}+\mathscr{S}]$, it follows that $\mathscr{B}(W \mid \mathscr{L}) \neq 0$ for W in $[\mathscr{T}+\mathscr{S}]\backslash\mathscr{S}$ iff (7) holds.

Define

$$\mathscr{A}(\mathscr{L}) = \{W \in [\mathscr{T} + \mathscr{S}] \setminus \mathscr{S} \colon \mathscr{B}(W \mid \mathscr{L}) \text{ is nonempty}\}.$$

LaMotte (1982) has given a stepwise algorithm to characterize admissible linear estimators. To formalize this procedure let us introduce the following notation.

For $i = 1, 2, 3$, define the following families of affine sets in $\mathscr{M}_{3\times 3}$:

$$\mathscr{C}^{(0)} = \{\mathscr{M}_{3\times 3}\},$$

$$\mathscr{C}^{(i)} = \{\mathscr{B}(W \mid \mathscr{L}) \colon \mathscr{L} \in \mathscr{C}^{(i-1)},\ W \in \mathscr{A}(\mathscr{L})\}.$$

The set of admissible linear estimators of $C'\theta$ can be described as

$$\{L'Z \colon \{L\} \in \mathscr{C}^{(i)} \text{ for some } i = 1, 2, 3\}$$

(see also Zontek (1987) or Klonecki and Zontek (1985)).

2. A CONSTRUCTION OF ADMISSIBLE LINEAR ESTIMATORS

In this section we use the above-described stepwise procedure to construct all admissible linear estimators of θ in the linear model given by (3) and (4), that is, all admissible quadratic estimators of θ in the linear model given by (1).

It is easy to see that each point $W = (W_1, W_2) \in \mathscr{W}$ is uniquely determined by a matrix $\Delta = \{d_{ij}\} \in \mathscr{S}_3$. Indeed,

$$W_1 = W_1(\Delta) = 2T \begin{bmatrix} v_{11} & 0 & 0 \\ 0 & v_{22} & 0 \\ 0 & 0 & v_{33} \end{bmatrix},$$

where $v_{11} = v_{22} + (d_{12} + d_{21}) b_1 b_2 + (d_{13} + d_{31}) b_1$, $v_{22} = d_{22} b_2^2 + (d_{23} + d_{32}) b_2 + d_{33}$ and $v_{33} = d_{33}$, and

$$W_2 = W_2(\Delta) = TH'\Delta HT.$$

The set $[\mathscr{T}]$ corresponds to matrices Δ's from the set of all n.n.d. matrices in $\mathscr{S}_3$ with nonnegative entries.

2.1. The first type of admissible estimators (three steps). In the first step we take

$$\Delta_1 = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad d_{11} > 0.$$

Simple calculations show that

(8) $$\mathscr{L}_1 = \mathscr{B}(\mathscr{M}_{3\times 3} \mid W(\Delta_1)) = \{M_1 C + \Pi_1 X \colon X \in \mathscr{M}_{3\times 3}\},$$

where

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Pi_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Note that M_1 does not depend on d_{11}, so we would take for example $d_{11} = 1$.

The set $\mathscr{S}(\mathscr{L}_1)$ corresponds to the set

$$\left\{ \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & 0 & 0 \\ s_{13} & 0 & 0 \end{bmatrix} : s_{11}, s_{12}, s_{13} \in \mathscr{R} \right\}, \tag{9}$$

and $\mathscr{A}(\mathscr{L}_1)$ corresponds to the set

$$\left\{ \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & d_{22} & d_{32} \\ s_{13} & d_{32} & d_{33} \end{bmatrix} : \begin{array}{l} s_{11}, s_{12}, s_{13} \in \mathscr{R},\ d_{22}, d_{32}, d_{33} \geqslant 0, \\ d_{32}^2 \leqslant d_{22} d_{33},\ d_{22} + d_{33} > 0 \end{array} \right\}. \tag{10}$$

In the second step we take

$$\Delta_2 = \begin{bmatrix} x & x & x \\ x & d_{22} & 0 \\ x & 0 & 0 \end{bmatrix}, \quad d_{22} > 0,$$

where instead of symbols x we can put any numbers. As before, without loss of generality we can take $d_{22} = 1$. Then

$$\mathscr{L}_2 = \mathscr{B}(\mathscr{L}_1 \mid W(\Delta_2)) = \{M_2 C + \Pi_2 X\colon\ X \in \mathscr{M}_{3\times 3}\},$$

where

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t_2/(2+t_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The set $\mathscr{S}(\mathscr{L}_2)$ corresponds to the set

$$\left\{ \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & 0 \\ s_{13} & 0 & 0 \end{bmatrix} : s_{11}, s_{12}, s_{13}, s_{22} \in \mathscr{R} \right\}$$

and $\mathscr{A}(\mathscr{L}_2)$ corresponds to the set

$$\left\{ \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & d_{32} \\ s_{13} & d_{32} & d_{33} \end{bmatrix} : s_{11}, s_{12}, s_{13}, s_{22} \in \mathscr{R},\ d_{32} \geqslant 0,\ d_{33} > 0 \right\}.$$

In the last step we take

$$\Delta_3 = \begin{bmatrix} x & x & x \\ x & x & d_{32} \\ x & d_{32} & 1 \end{bmatrix}, \quad d_{32} \geqslant 0.$$

Then

$$\mathscr{L}_3 = \mathscr{B}(\mathscr{L}_2 \mid W(\Delta_3)) = \{M_3 C\},$$

where

$$M_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \dfrac{t_2}{2+t_2} & 0 \\ 0 & \dfrac{2(b_2 d_{32}+1)t_2}{(2+t_2)(2+t_3)} & \dfrac{t_3}{2+t_3} \end{bmatrix}.$$

One can verify that the matrix $M_3 C$ is the unique solution of the following equation:

$$\sum_{i=1}^{3} \Pi_{i-1}[W_1(\Delta_i)+W_2(\Delta_i)]L = \sum_{i=1}^{3} \Pi_{i-1} W_2(\Delta_i)C, \tag{11}$$

where

$$\Pi_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

2.2. The second type of admissible estimators (two steps). There are two situations. In the first step we can take as before

$$\Delta_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Consequently, the sets $\mathscr{L}_1 = \mathscr{B}(\mathscr{M}_{3\times 3} \mid W(\Delta_1))$, $\mathscr{S}(\mathscr{L}_1)$ and $\mathscr{A}(\mathscr{L}_1)$ are given by (8), (9) and (10), respectively.

In the second step we choose Δ_2 in the following form:

$$\Delta_2 = \begin{bmatrix} x & x & x \\ x & d_{22} & d_{32} \\ x & d_{32} & 1 \end{bmatrix}, \qquad d_{32} \geqslant 0,\ d_{22} \geqslant 0,\ d_{32}^2 \leqslant d_{22}.$$

Then

$$\mathscr{L}_2 = \mathscr{B}(\mathscr{L}_1 \mid W(\Delta_2)) = \{M_2 C\},$$

where

$$M_2 = \frac{1}{w}\begin{bmatrix} w & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix},$$

and

$$m_{22} = t_2[2+4d_{32}b_2+b_2^2(-d_{32}^2 t_3+d_{22}(2+t_3))],$$

$$m_{23} = 2(1+b_2 d_{32})t_3,$$

$$m_{32} = 2(1+b_2 d_{32})[1+b_2(b_2 d_{22}+2d_{32})]t_2,$$

$$m_{33} = [2+4d_{32}b_2+b_2^2(-d_{32}^2 t_2+d_{22}(2+t_2))]\,t_3,$$

$$w = 2(1+2b_2 d_{32})(2+t_2+t_3)+b_2^2[-d_{32}^2 t_2 t_3+d_{22}(2+t_2)(2+t_3)].$$

Similarly to the previous situation, the resulting matrix defining admissible linear estimator is the unique solution of the equation

$$\sum_{i=1}^{2} \Pi_{i-1}[W_1(\Delta_i)+W_2(\Delta_i)]\,L = \sum_{i=1}^{2} \Pi_{i-1} W_2(\Delta_i)\,C. \tag{12}$$

There is another way of constructing the admissible estimators in two steps. In the first step we take

$$\Delta_1 = \begin{bmatrix} d_{11} & d_{12} & 0 \\ d_{12} & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad d_{11} \geqslant 0,\ d_{12} \geqslant 0,\ d_{12}^2 \leqslant d_{11}.$$

Then

$$\mathscr{L}_1 = \mathscr{B}(\mathscr{M}_{3\times 3} \mid W(\Delta_1)) = \{M_1 C+\Pi_1 X\colon\ X \in \mathscr{M}_{3\times 3}\},$$

where

$$M_1 = \frac{1}{w}\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

and

$$m_{11} = t_1[2b_2^2+4b_1 b_2 d_{12}+b_1^2(-d_{12}^2 t_2+d_{11}(2+t_2))],$$

$$m_{12} = 2b_2(b_2+b_1 d_{12})t_2,$$

$$m_{21} = 2(b_2+b_1 d_{12})(b_2+2b_1 d_{12})t_1,$$

$$m_{22} = [2b_2(b_2+2b_1 d_{12})+b_1^2(d_{11}-d_{12}^2)t_1]\,t_2,$$

$$w = 2b_2(b_2+2b_1 d_{12})(2+t_1+t_2)+b_1^2 t_1[-d_{12}^2 t_2+d_{11}(2+t_2)],$$

while

$$\Pi_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In the second step we take

$$\Delta_2 = \begin{bmatrix} x & x & d_{31} \\ x & x & d_{32} \\ d_{31} & d_{32} & 1 \end{bmatrix}, \qquad d_{31} \geqslant 0,\ d_{32} \geqslant 0.$$

Then

$$\mathscr{L}_2 = \mathscr{B}(\mathscr{L}_1 \mid W(\Delta_2)) = \{M_2 C\},$$

where

$$M_2 = \frac{1}{w}\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ m_{31} & m_{32} & wt_3/(2+t_3) \end{bmatrix},$$

and

$$m_{31} = \frac{(1+b_1 d_{31}+b_2 d_{32})(w-m_{11})t_1-(1+b_2 d_{32})m_{21}t_2}{2+t_3},$$

$$m_{32} = \frac{-(1+b_1 d_{31}+b_2 d_{32})m_{21}t_1+(1+b_2 d_{32})(w-m_{22})t_2}{2+t_3},$$

$$w = 2b_2(b_2+2b_1 d_{12})(2+t_1+t_2)+b_1^2 t_1[-d_{12}^2 t_2+d_{11}(2+t_2)].$$

Again a matrix being the unique solution of the equation

$$\sum_{i=1}^{2} \Pi_{i-1}[W_1(\Delta_i)+W_2(\Delta_i)]L = \sum_{i=1}^{2} \Pi_{i-1} W_2(\Delta_i)C \tag{13}$$

defines an admissible estimator of θ.

Note that this equation is a copy of (12) but matrices Π_1, Δ_1 and Δ_2 are different.

2.3. The third type of admissible estimators (one step). As Δ_1 we take a 3×3 n.n.d. matrix with nonnegative entries and with $d_{33}=1$. Then an estimator $L'Z$ locally best among $\mathcal{M}_{3\times 3}$ at $W(\Delta_1)$ is unique and, of course, admissible for θ. So L is the unique solution of the equation

$$[W_1(\Delta_1)+W_2(\Delta_1)]L = W_2(\Delta_1)C. \tag{14}$$

2.4. The main results. LaMotte's theorem implies that there is no other possibility of constructing the admissible estimators. Therefore we get the following characterization of admissible linear estimators of θ.

THEOREM 1. *An estimator $L'Z$ is admissible for θ if and only if L is the solution of* (11), (12), (13) *or* (14).

The model described by (3) and (4) is regular in the sense of Zontek (1987) (see also Klonecki and Zontek (1985)). For regular models, admissible estimators of a linear parametric function can be characterized by admissible estimators of the vector of parameters. So we have the following theorem where f is a 3-dimensional vector.

THEOREM 2. *An estimator $g'Z$ is admissible for $f'\theta$ if and only if*

$$g \in \{Lf\colon\ L'Z \text{ is admissible for } \theta\}.$$

This method of constructing the admissible estimators can be applied to estimation of the mean squared error of a linear estimator $l'Y$ of μ, where

$l \in \mathscr{R}^{nJ}$, in the model (1). The mean squared error is a linear combination of μ^2, σ^2 and σ_e^2 for f given by

$$f = \big((l' \mathbf{1}_{nJ} - 1)^2,\ l'(I_n \otimes KK')\,l,\ l'l\big)'.$$

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