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A CENTRAL LIMIT THEOREM FOR MULTIVARIATE STRONGLY MIXING RANDOM FIELDS

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Abstract. In this paper we extend a theorem of Bradley under interlaced mixing and strong mixing conditions. More precisely, we study the asymptotic normality of the normalized partial sum of an α-mixing strictly stationary random field of random vectors, in the presence of another dependence assumption.

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1. INTRODUCTION

This paper presents a central limit theorem for strictly stationary random fields of random vectors satisfying a certain strong mixing condition, in the presence of another dependence assumption involving the maximal correlation coefficient. This result is actually an extension of the central limit theorem for real-valued random fields of Corollary 29.33 from Bradley [4].

For the clarity of the main result, relevant definitions and notation will be given in the following.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. For any two σ-fields $\mathcal{A}$, $\mathcal{B} \subseteq \mathcal{F}$, define the strong mixing coefficient

$$\alpha(\mathcal{A}, \mathcal{B}) = \sup_{A \in \mathcal{A}, B \in \mathcal{B}} |P(A \cap B) - P(A)P(B)|,$$

and the maximal coefficient of correlation

$$\rho(\mathcal{A}, \mathcal{B}) = \sup |\mathrm{Corr}(f, g)|, \quad f \in L^2_{\mathrm{real}}(\mathcal{A}),\ g \in L^2_{\mathrm{real}}(\mathcal{B}).$$

Suppose d and m are each a positive integer, and $X := (X_{\mathbf{k}}, \mathbf{k} \in \mathbb{Z}^d)$ is a strictly stationary random field with the random variables $X_{\mathbf{k}}$ being $\mathbb{R}^m$-valued. If all the coordinates of the m-dimensional random variable $X_{\mathbf{k}}$ have finite second moments, then the $m \times m$ covariance matrix of $X_{\mathbf{k}}$ will be denoted by $\Sigma_{X_{\mathbf{k}}}$.

Throughout this paper, for given positive integers d and m, we will use the boldface notation $\mathbf{0} := (0, 0, \ldots, 0)$ to denote the origin in $\mathbb{Z}^d$; 0_m to denote the origin in $\mathbb{R}^m$, and I_m to denote the $m \times m$ identity matrix.

In this context, for each positive integer n, define the quantities:

$$\alpha(n) := \alpha(X, n) := \sup \alpha\big(\sigma(X_{\mathbf{k}}, \mathbf{k} \in Q), \sigma(X_{\mathbf{k}}, \mathbf{k} \in S)\big),$$

where the supremum is taken over all pairs of nonempty, disjoint sets Q, $S \subset \mathbb{Z}^d$ with the following property: There exist $u \in \{1, 2, \ldots, d\}$ and $j \in \mathbb{Z}$ such that $Q \subset \{\mathbf{k} := (k_1, k_2, \ldots, k_d) \in \mathbb{Z}^d : k_u \leqslant j\}$ and $S \subset \{\mathbf{k} := (k_1, k_2, \ldots, k_d) \in \mathbb{Z}^d : k_u \geqslant j + n\}$.

The random field $X := (X_{\mathbf{k}}, \mathbf{k} \in \mathbb{Z}^d)$ is said to be *strongly mixing* (or *α-mixing*) if $\alpha(n) \to 0$ as $n \to \infty$.

Also, for each positive integer n define the quantity:

$$\rho'(n) := \rho'(X, n) := \sup \rho\big(\sigma(X_{\mathbf{k}}, \mathbf{k} \in Q), \sigma(X_{\mathbf{k}}, \mathbf{k} \in S)\big),$$

where the supremum is taken over all pairs of nonempty, finite disjoint sets $Q, S \subset \mathbb{Z}^d$ with the following property: There exist $u \in \{1, 2, \ldots, d\}$ and nonempty disjoint sets A, $B \subset \mathbb{Z}$ with $\operatorname{dist}(A, B) := \min_{a \in A, b \in B} |a - b| \geqslant n$, such that $Q \subset \{\mathbf{k} := (k_1, k_2, \ldots, k_d) \in \mathbb{Z}^d : k_u \in A\}$ and $S \subset \{\mathbf{k} := (k_1, k_2, \ldots, k_d) \in \mathbb{Z}^d : k_u \in B\}$.

The random field $X := (X_{\mathbf{k}}, \mathbf{k} \in \mathbb{Z}^d)$ is said to be *ρ'-mixing* if $\rho'(n) \to 0$ as $n \to \infty$.

Again, suppose d and m are each a positive integer, and $X := (X_{\mathbf{k}}, \mathbf{k} \in \mathbb{Z}^d)$ is a strictly stationary random field with the random variables $X_{\mathbf{k}}$ being $\mathbb{R}^m$-valued. For any $\mathbf{L} := (L_1, L_2, \ldots, L_d) \in \mathbb{N}^d$, define the "rectangular sum":

$$S_{\mathbf{L}} = S(X, \mathbf{L}) := \sum_{\mathbf{k}} X_{\mathbf{k}}, \tag{1.1}$$

where the sum is taken over all d-tuples $\mathbf{k} := (k_1, k_2, \ldots, k_d) \in \mathbb{N}^d$ such that $1 \leqslant k_u \leqslant L_u$ for all $u \in \{1, 2, \ldots, d\}$.

Also, for any given $\mathbf{L} \in \mathbb{N}^d$, let us denote the product of its components by

$$\prod(\mathbf{L}) := L_1 \cdot L_2 \cdot \ldots \cdot L_d. \tag{1.2}$$

Therefore, by definition (1.1), $S(X, \mathbf{L})$ is the sum of $\prod(\mathbf{L})$ m-dimensional random vectors $X_{\mathbf{k}}$.

THEOREM 1.1. *Suppose d and m are each a positive integer. Suppose $X := (X_{\mathbf{k}}, \mathbf{k} \in \mathbb{Z}^d)$ is a strictly stationary random field where for a given $\mathbf{k} \in \mathbb{Z}^d$, the $\mathbb{R}^m$-valued random variable, $X_{\mathbf{k}}$, satisfies the following properties:*

$$EX_{\mathbf{0}} = 0_m \tag{1.3}$$

and

$$E\,\|X_{\mathbf{0}}\|_2^2 < \infty. \tag{1.4}$$

Suppose that

$$\rho'(1) < 1 \qquad \text{and} \qquad \alpha(n) \to 0 \text{ as } n \to \infty. \tag{1.5}$$

Assume also that the covariance matrix of the $\mathbb{R}^m$-valued random variable $X_{\mathbf{0}}$ is nonsingular. Then we have the following two properties:

(I) *For each $\mathbf{L} \in \mathbb{N}^d$, the covariance matrix $\Sigma_{S(X,\mathbf{L})}$ is nonsingular.*

(II) *As $\|\mathbf{L}\|_2 \to \infty$,*

$$\Sigma_{S(X,\mathbf{L})}^{-1/2} S(X,\mathbf{L}) \Rightarrow N(0_m, I_m).$$

Theorem 1.1 extends a result of Bradley, specified as Corollary 29.33 in [4], which deals with the special case of strictly stationary random fields of real-valued random variables.

For the special case of strictly stationary random sequences of real-valued random variables, Theorem 1.1 was already proved by Peligrad in [6]. This result was later generalized by Utev and Peligrad in [7] to a weak invariance principle for (not necessarily stationary) triangular arrays of sequences of real-valued random variables under a Lindeberg condition and analogs of the mixing assumptions in Theorem 1.1.

For strictly stationary random fields of $\mathbb{R}^m$-valued random variables under quite different dependence assumptions, a central limit theorem somewhat like Theorem 1.1 was proved by Bulinski and Kryzhanovskaya in [5].

2. PRELIMINARIES

In the following, we collect the background results we would need for the proof of Theorem 1.1.

First, let us mention that for $m \times 1$ vectors $\mathbf{a}$, $\mathbf{b} \in \mathbb{R}^m$, the "dot product" notation will be used: $\mathbf{a} \cdot \mathbf{b} = \mathbf{a}^t \mathbf{b}$.

For real numbers $r_1, r_2, \ldots, r_m$, let $[\mathrm{diag}(r_1, r_2, \ldots, r_m)]$ denote the $m \times m$ diagonal matrix whose diagonal entries are $r_1, r_2, \ldots, r_m$.

REMARK 2.1. *Let $G := (g_{ij},\ 1 \leqslant i,\ j \leqslant m)$ be a symmetric, nonnegative definite $m \times m$ matrix. Then:*

(I) *$G = PDP^t$, where P is an orthogonal matrix, $D = [\mathrm{diag}(d_1, d_2, \ldots, d_m)]$, and the eigenvalues of G are $d_1, d_2, \ldots, d_m$ with $0 \leqslant d_1 \leqslant d_2 \leqslant \ldots \leqslant d_m$.*

(II) *Representing the elements of $\mathbb{R}^m$ as $m \times 1$ column vectors, we have the following properties:*

$$\text{(i)} \qquad d_1 = \inf_{\{\mathbf{a} \in \mathbb{R}^m : \|\mathbf{a}\|_2 = 1\}} \mathbf{a}^t G \mathbf{a},$$

(ii)
$$d_m = \sup_{\{\mathbf{a}\in\mathbb{R}^m : \|\mathbf{a}\|_2=1\}} \mathbf{a}^t G\mathbf{a},$$

(iii)
$$\forall i, j \in \{1, 2, \ldots, m\}, \quad |g_{ij}| \leqslant d_m.$$

(III) *There exists a unique symmetric, nonnegative definite* $m \times m$ *matrix* B *such that* $B^2 = G$. *Note that* $B := G^{1/2} = PD^{1/2}P^t$, *where*

$$D^{1/2} := \operatorname{diag}(\sqrt{d_1}, \sqrt{d_2}, \ldots, \sqrt{d_m}).$$

(IV) *In addition, if* G (*and hence* $G^{1/2}$) *is nonsingular, then* $(G^{1/2})^{-1} := G^{-1/2} = PD^{-1/2}P^t$, *where*

$$D^{-1/2} := \operatorname{diag}(d_1^{-1/2}, d_2^{-1/2}, \ldots, d_m^{-1/2}).$$

Of course, $G^{-1/2}$ *is symmetric and positive definite.*

REMARK 2.2. *Assume that* W *is an* $m \times 1$ *random vector with* $EW_i = 0$ *and* $EW_i^2 < \infty$ *for each* $i \in \{1, 2, \ldots, m\}$. *Then we have the following properties*:

(I) *The covariance matrix* Σ_W *is symmetric and nonnegative definite.*

(II) *Letting* $d_1 \leqslant d_2 \leqslant \ldots \leqslant d_m$ *denote the eigenvalues of the covariance matrix* Σ_W, *the items* (i) *and* (ii) *of Remark* 2.1 *take the following form*:

(i′)
$$d_1 = \inf_{\{\mathbf{a}\in\mathbb{R}^m : \|\mathbf{a}\|_2=1\}} E\,(\mathbf{a}\cdot W)^2$$

and

(ii′)
$$d_m = \sup_{\{\mathbf{a}\in\mathbb{R}^m : \|\mathbf{a}\|_2=1\}} E\,(\mathbf{a}\cdot W)^2.$$

CLAIM 2.1. Let W be the $m \times 1$ random vector defined in Remark 2.2. Let its covariance matrix Σ_W be symmetric and positive definite. Then for all $\mathbf{a} \in \mathbb{R}^m - \{0_m\}$, $\mathbf{a} \cdot W$ is a nondegenerate random variable.

REMARK 2.3. *Suppose* c_1 *and* c_2 *are positive numbers*; A_1, A_2, $A_3, \ldots$ *is a sequence of symmetric, positive definite* $m \times m$ *matrices whose eigenvalues are all bounded within the interval* $[c_1, c_2]$; A *is an* $m \times m$ *matrix*; *and* $A_n \to A$ *as* $n \to \infty$. *Then* A *is a symmetric, positive definite matrix whose eigenvalues are bounded within the interval* $[c_1, c_2]$, *and as* $n \to \infty$ *we have* $A_n^r \to A^r$ *for each* $r \in \{1/2, -1, -1/2\}$.

3. PROOF OF THEOREM 1.1

Let Σ_{X_0} denote the $m \times m$ covariance matrix of the random vector X_0. Let $d_1, d_2, \ldots, d_m$ be the eigenvalues of the covariance matrix Σ_{X_0} with the property that $d_1 \leqslant d_2 \leqslant \ldots \leqslant d_m$. $\Sigma_{X_\mathbf{0}}$ is symmetric and nonnegative definite and, by hypothesis, it is also nonsingular. It follows that

$$0 < d_1 \leqslant d_2 \leqslant \ldots \leqslant d_m < \infty, \tag{3.1}$$

and hence $\Sigma_{X_\mathbf{0}}$ is symmetric and positive definite.

Let us now represent $\Sigma_{X_\mathbf{0}} = PDP^t$, where P is an orthogonal matrix and $D = [\mathrm{diag}(d_1, d_2, \ldots, d_m)]$. Note that, by (1.3), (1.4), and Claim 2.1, for all $\mathbf{a} \in \mathbb{R}^m - \{0_m\}$, $\mathbf{a} \cdot X_0$ is a nondegenerate random variable.

Proof of (I). Suppose $\mathbf{L} \in \mathbb{N}^d$. Let $\Sigma_{S(X,\mathbf{L})/\sqrt{\prod(\mathbf{L})}}$ denote the $m \times m$ covariance matrix of the $\mathbb{R}^m$-valued random vector $S(X, \mathbf{L}) / \sqrt{\prod(\mathbf{L})}$. Let us notice that $\Sigma_{S(X,\mathbf{L})} = \prod(\mathbf{L})\Sigma_{S(X,\mathbf{L})/\sqrt{\prod(\mathbf{L})}}$.

Let us now define the positive constant

$$C := \left(1 + \rho'(1)\right)^d / \left(1 - \rho'(1)\right)^d. \tag{3.2}$$

CLAIM 3.1. For each $\mathbf{L} \in \mathbb{N}^d$, the $m \times m$ covariance matrix $\Sigma_{S(X,\mathbf{L})/\sqrt{\prod(\mathbf{L})}}$ is nonsingular and its eigenvalues are bounded below by $C^{-1}d_1 > 0$ and bounded above by $Cd_m < \infty$, where C is the positive constant defined in (3.2). In addition, every entry of the covariance matrix $\Sigma_{S(X,\mathbf{L})/\sqrt{\prod(\mathbf{L})}}$ is bounded in absolute value by Cd_m.

Proof. Suppose $\mathbf{a} \in \mathbb{R}^m$ such that $\|\mathbf{a}\|_2 = 1$. By Remark 2.2, part (II), followed by (3.1), we obtain $0 < d_1 \leqslant E\left(\mathbf{a} \cdot X_\mathbf{0}\right)^2 \leqslant d_m < \infty$.

Referring to (1.3)–(1.5) and (3.2), by Theorem 28.9 in [4], we have the following properties:

$$0 < C^{-1} < C < \infty \tag{3.3}$$

and

$$C^{-1} \cdot E\left(\mathbf{a} \cdot X_\mathbf{0}\right)^2 \leqslant E\left(\mathbf{a} \cdot \frac{S(X, \mathbf{L})}{\sqrt{\prod(\mathbf{L})}}\right)^2 \leqslant C \cdot E\left(\mathbf{a} \cdot X_\mathbf{0}\right)^2. \tag{3.4}$$

By (3.3), (1.4) and Claim 2.1, we obtain

$$0 < C^{-1} \cdot E\left(\mathbf{a} \cdot X_\mathbf{0}\right)^2 \leqslant E\left(\mathbf{a} \cdot \frac{S(X, \mathbf{L})}{\sqrt{\prod(\mathbf{L})}}\right)^2 \leqslant C \cdot E\left(\mathbf{a} \cdot X_\mathbf{0}\right)^2 < \infty. \tag{3.5}$$

By Remark 2.2, part (II), the inequalities (3.5) imply

$$0 < C^{-1}d_1 \leqslant E\left(\mathbf{a} \cdot \frac{S(X, \mathbf{L})}{\sqrt{\prod(\mathbf{L})}}\right)^2 \leqslant Cd_m < \infty. \tag{3.6}$$

Since $\mathbf{a} \in \mathbb{R}^m$ was arbitrary such that $\|\mathbf{a}\|_2 = 1$, we infer by Remark 2.2, part (II), that the eigenvalues of the covariance matrix $\Sigma_{S(X,\mathbf{L})/\sqrt{\Pi(\mathbf{L})}}$ are bounded below by $C^{-1}d_1 > 0$ and bounded above by $Cd_m < \infty$. Therefore, $\Sigma_{S(X,\mathbf{L})/\sqrt{\Pi(\mathbf{L})}}$ is a nonsingular matrix with every entry being bounded in absolute value by Cd_m. Therefore, the proof of Claim 3.1 is complete. ■

For a given $\mathbf{L} \in \mathbb{N}^d$, since $\Sigma_{S(X,\mathbf{L})/\sqrt{\Pi(\mathbf{L})}}$ is nonsingular by Claim 3.1, $\Sigma_{S(X,\mathbf{L})}$ is also nonsingular, and hence the proof of part (I) is complete.

Proof of (II). Let us now show the following:

CLAIM 3.2. For each $\mathbf{L} \in \mathbb{N}^d$, $\Sigma^{-1/2}_{S(X,\mathbf{L})} = \big(\prod(\mathbf{L})\big)^{-1/2}\Sigma^{-1/2}_{S(X,\mathbf{L})/\sqrt{\Pi(\mathbf{L})}}$.

Proof. Claim 3.2 follows simply from basic linear algebra properties and the trivial fact that $\Sigma_{S(X,\mathbf{L})} = \prod(\mathbf{L})\Sigma_{S(X,\mathbf{L})/\sqrt{\Pi(\mathbf{L})}}$. ■

By Claim 3.2, for $\mathbf{L} \in \mathbb{N}^d$ we obviously have:

$$\Sigma^{-1/2}_{S(X,\mathbf{L})}S(X,\mathbf{L}) = \big(\prod(\mathbf{L})\big)^{-1/2}\Sigma^{-1/2}_{S(X,\mathbf{L})/\sqrt{\Pi(\mathbf{L})}}\big(\prod(\mathbf{L})\big)^{1/2}\frac{S(X,\mathbf{L})}{\big(\prod(\mathbf{L})\big)^{1/2}} = \Sigma^{-1/2}_{S(X,\mathbf{L})/\sqrt{\Pi(\mathbf{L})}}\frac{S(X,\mathbf{L})}{\sqrt{\prod(\mathbf{L})}}. \tag{3.7}$$

Refer now to [4], Proposition A2906, part (III). Let $u \in \{1, 2, \dots, d\}$ be arbitrary but fixed. Let $\mathbf{L}^{(1)}, \mathbf{L}^{(2)}, \mathbf{L}^{(3)}, \dots$ be an arbitrary fixed sequence of elements of $\mathbb{N}^d$ such that for each $n \geqslant 1$, $L^{(n)}_u = n$ and $L^{(n)}_v \geqslant 1$ for all $v \in \{1, 2, \dots, d\} - \{u\}$.

With no loss of generality, we can permute the indices in the coordinate system of Z^d, in order to have $u = 1$, and therefore $L^{(n)}_1 = n$ for $n \geqslant 1$ and $L^{(n)}_v \geqslant 1$ for all $v \in \{2, \dots, d\}$. For each $n \geqslant 1$, let us represent

$$\mathbf{L}^{(n)} := (n, L^{(n)}_2, L^{(n)}_3, \dots, L^{(n)}_d). \tag{3.8}$$

Obviously, $\|\mathbf{L}^{(n)}\|_2 \to \infty$ as $n \to \infty$.

To complete the proof of part (II), and hence the proof of the theorem, by (3.7), it suffices to show that

$$\Sigma^{-1/2}_{S(X,\mathbf{L}^{(n)})/\sqrt{\Pi(\mathbf{L}^{(n)})}}\frac{S(X,\mathbf{L}^{(n)})}{\sqrt{\prod(\mathbf{L}^{(n)})}} \Rightarrow N(0_m, I_m) \quad \text{as } n \to \infty. \tag{3.9}$$

Refer to [2], Theorem 2.6. Let Q be an arbitrary infinite set, $Q \subseteq \mathbb{N}$. It suffices to show that there exists an infinite set $T \subseteq Q$ such that

$$\Sigma^{-1/2}_{S(X,\mathbf{L}^{(n)})/\sqrt{\Pi(\mathbf{L}^{(n)})}}\frac{S(X,\mathbf{L}^{(n)})}{\sqrt{\prod(\mathbf{L}^{(n)})}} \Rightarrow N(0_m, I_m) \quad \text{as } n \to \infty,\ n \in T. \tag{3.10}$$

By Claim 3.1, followed by the compactness argument, for the infinite set $Q \subseteq \mathbb{N}$, there exist an infinite subset $T \subseteq Q$ and an $m \times m$ matrix Σ such that

$$\Sigma_{S(X,\mathbf{L}^{(\mathbf{n})})/\sqrt{\Pi(\mathbf{L}^{(\mathbf{n})})}} \to \Sigma \quad \text{as } n \to \infty,\ n \in T. \tag{3.11}$$

The $m \times m$ matrix Σ is nonsingular by Remark 2.3, and its eigenvalues are bounded below by $C^{-1}d_1 > 0$. Obviously, we obtain

(3.12)

$$\Sigma^{-1/2}\Sigma_{S(X,\mathbf{L}^{(\mathbf{n})})/\sqrt{\Pi(\mathbf{L}^{(\mathbf{n})})}}\Sigma^{-1/2} \to \Sigma^{-1/2}\Sigma\Sigma^{-1/2} = I_m \quad \text{as } n \to \infty,\ n \in T.$$

As a consequence, for every $\mathbf{a} \in \mathbb{R}^m$, $\mathbf{a} \neq 0_m$, we obtain the equivalence of the variance terms:

$$E\left(\mathbf{a} \cdot \Sigma^{-1/2}\frac{S(X,\mathbf{L}^{(\mathbf{n})})}{\sqrt{\prod(\mathbf{L}^{(\mathbf{n})})}}\right)^2 \to \|\mathbf{a}\|_2^2 \quad \text{as } n \to \infty,\ n \in T. \tag{3.13}$$

Now, $\mathbf{a} \cdot \Sigma^{-1/2}S(X,\mathbf{L}^{(\mathbf{n})})/\sqrt{\prod(\mathbf{L}^{(\mathbf{n})})}$ is a real-valued random variable, and therefore, by [4], Corollary 29.33, it follows that

$$\frac{\mathbf{a} \cdot \Sigma^{-1/2}S(X,\mathbf{L}^{(\mathbf{n})})\big(\sqrt{\prod(\mathbf{L}^{(\mathbf{n})})}\big)^{-1}}{\big\|\mathbf{a} \cdot \Sigma^{-1/2}S(X,\mathbf{L}^{(\mathbf{n})})\big(\sqrt{\prod(\mathbf{L}^{(\mathbf{n})})}\big)^{-1}\big\|_2} \Rightarrow N(0,1) \quad \text{as } n \to \infty. \tag{3.14}$$

By (3.13) and (3.14), followed by Slutski's theorem we obtain the following:

$$\mathbf{a} \cdot \Sigma^{-1/2}\frac{S(X,\mathbf{L}^{(\mathbf{n})})}{\sqrt{\prod(\mathbf{L}^{(\mathbf{n})})}} \Rightarrow N(0,\|\mathbf{a}\|_2^2) \quad \text{as } n \to \infty,\ n \in T. \tag{3.15}$$

Since $\mathbf{a} \in \mathbb{R}^m$ was arbitrary, as a consequence, (3.15) is equivalent to

$$\Sigma^{-1/2}\frac{S(X,\mathbf{L}^{(\mathbf{n})})}{\sqrt{\prod(\mathbf{L}^{(\mathbf{n})})}} \Rightarrow N(0_m, I_m) \quad \text{as } n \to \infty,\ n \in T. \tag{3.16}$$

By (3.11), (3.16) and the multivariate Slutski theorem, we derive that

$$\Sigma^{-1/2}_{S(X,\mathbf{L}^{(\mathbf{n})})/\sqrt{\Pi(\mathbf{L}^{(\mathbf{n})})}}\frac{S(X,\mathbf{L}^{(\mathbf{n})})}{\sqrt{\prod(\mathbf{L}^{(\mathbf{n})})}} \Rightarrow N(0_m, I_m) \quad \text{as } n \to \infty,\ n \in T.$$

Therefore, (3.10) holds, and as a consequence, (3.9) holds too. Hence, the proof of Theorem 1.1 is complete.

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