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TRACTABILITY OF MULTI-PARAMETRIC EULER AND WIENER INTEGRATED PROCESSES

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Abstract: We study average case approximation of Euler and Wiener integrated processes of d variables which are almost surely r_k-times continuously differentiable with respect to the k-th variable and $0 \le r_k \le r_{k+1}$. Let $n(\varepsilon, d)$ denote the minimal number of continuous linear functionals which is needed to find an algorithm that uses n such functionals and whose average case error improves the average case error of the zero algorithm by a factor ε. Strong polynomial tractability means that there are nonnegative numbers C and p such that

$$n(\varepsilon, d) \le C\varepsilon^{-p} \quad \text{for all } d \in \mathbb{N} = \{1, 2, \dots\}, \text{ and } \varepsilon \in (0, 1).$$

We prove that the Wiener process is much more difficult to approximate than the Euler process. Namely, strong polynomial tractability holds for the Euler case iff

$$\liminf_{k\to\infty} \frac{r_k}{\ln k} > \frac{1}{2\ln 3},$$

whereas it holds for the Wiener case iff

$$\liminf_{k\to\infty} \frac{r_k}{k^s} > 0 \quad \text{for some } s > \tfrac{1}{2}.$$

Other types of tractability are also studied.

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