PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 32, Fasc. 2 (2012), pp. 255–279

ON A BAHADUR–KIEFER REPRESENTATION OF VON MISES STATISTIC TYPE FOR INTERMEDIATE SAMPLE QUANTILES

Nadezhda Gribkova
Roelof Helmers

Abstract: We investigate a Bahadur–Kiefer type representation for the p_n-th empirical quantile corresponding to a sample of n i.i.d. random variables when $p_n \in (0,1)$ is a sequence which, in particular, may tend to zero or one, i.e., we consider the case of intermediate sample quantiles. We obtain an 'in probability' version of the Bahadur–Kiefer type representation for a k_n-th order statistic when $r_n = k_n \wedge (n - k_n) \to \infty$, $n \to \infty$, without any restrictions of the rate at which r_n tends to infinity. We give a bound for the remainder term in the representation with probability $1 - O(r_n^{-c})$ for arbitrary $c > 0$. We obtain also an 'almost sure' version under the additional assumption that $\log n/r_n \to 0$ as $n \to \infty$.

Finally, we establish a Bahadur–Kiefer type representation for the sum of order statistics lying between the population p_n-quantile and the corresponding intermediate sample quantile by a von Mises type statistic approximation, especially useful in establishing second order approximations for slightly trimmed sums.

2000 AMS Mathematics Subject Classification: Primary: 62G30, 62G32; Secondary: 62G20, 62E20.

Keywords and phrases: Bahadur–Kiefer type representation, intermediate sample quantiles, Bahadur–Kiefer processes, empirical processes, quantile processes, von Mises statistic type approximation, slightly trimmed sum.