PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 36, Fasc. 2 (2016), pp. 311–333

A MAXIMAL INEQUALITY FOR STOCHASTIC INTEGRALS

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Abstract: Assume that X is a *càdlàg*, real-valued martingale starting from zero, H is a predictable process with values in $[-1,1]$ and $Y=\int H\,dX$. This article contains the proofs of the following inequalities:

(i) If X has continuous paths, then

$$\mathbb{P}(\sup_{t\geq 0} Y_t \geq 1) \leq 2\mathbb{E}\sup_{t\geq 0} X_t,$$

where the constant 2 is the best possible.

(ii) If X is arbitrary, then

$$\mathbb{P}(\sup_{t\geq 0} Y_t \geq 1) \leq c\mathbb{E}\sup_{t\geq 0} X_t,$$

where $c = 3.0446...$ is the unique positive number satisfying the equation $3c^4 - 8c^3 - 32 = 0$. This constant is the best possible.

2000 AMS Mathematics Subject Classification: Primary: 60G42; Secondary: 60G44.

Keywords and phrases: Martingale, sharp inequality.

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