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RUIN PROBABILITIES FOR TWO COLLABORATING INSURANCE COMPANIES

BY

ZBIGNIEW MICHNA (WROCŁAW)

Abstract. We find a formula for the supremum distribution of spectrally positive or negative Lévy processes with a broken linear drift. This gives formulas for ruin probabilities if two insurance companies (or two branches of the same company) divide between them both claims and premia in some specified proportions or if the premium rate for a given insurance portfolio is changed at a certain time. As an example we consider a gamma Lévy process, an α-stable Lévy process and Brownian motion. Moreover we obtain identities for the Laplace transform of the distribution of the supremum of Lévy processes with a randomly broken drift (random time of the premium rate change) and on random intervals (random time when the insurance portfolio is closed).

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1. INTRODUCTION

In this paper we study the supremum distribution of a spectrally positive or negative Lévy process with a piecewise linear drift. We find exact formulas for the distribution of the supremum which are expressed by one-dimensional densities of a given Lévy process. The results can be applied to find ruin probabilities if two insurance companies (insurer and reinsurer or two branches of the same company) divide between them both claims and premia in some specified proportions (proportional reinsurance) and the surplus is modeled by spectrally asymmetric Lévy processes which are now commonly used in insurance (see e.g. Furrer et al. [13]). The formulas can also be employed for the ruin probability of an insurance company or branch when the premium rate is changed at a certain time (e.g. the law regulations are changed in car insurance). Moreover the formulas can be used for a two-node tandem queue (see Lieshout and Mandjes [16]).

Avram et al. [1] investigate a spectrally positive Lévy process with a broken drift (reduction of the risk problem to one dimension) and they find the double Laplace transform of the infinite time survival probability. As an example they obtain an explicit analytical representation of the infinite time survival probability if the claims are exponentially distributed (the compound Poisson process with exponentially distributed claims). In Avram et al. [2] a related problem is investigated where the accumulated claim amount is modeled by a Lévy process that admits negative exponential moments. They find exact formulas for ruin probabilities expressed by ordinary ruin probabilities when the accumulated claim amount process is a spectrally negative Lévy process or a compound Poisson process with exponential claims. Additionally they find the asymptotic behavior of ruin probabilities under the Cramér assumption. In Foss et al. [12] the same problem as in Avram et al. [2] is investigated but subexponential claims are admitted and the asymptotic behavior of ruin probabilities for a finite and an infinite time horizon is found. Besides exact formulas and asymptotics there are other ways to approximate ruin probability: see e.g. Burnecki et al. [5] where De Vylder's idea is employed to estimate ruin probability in the model with claims and premia shared by two insurance companies.

In the models analyzed in this contribution we assume that the accumulated claim amount process is a spectrally positive or a spectrally negative Lévy process with one-dimensional density functions. We find exact formulas for ruin probabilities expressed by one-dimensional densities of the underlying Lévy process. The main difference between our models and those of Avram et al. [2] is that we admit heavy tailed claims and we provide explicit formulas for ruin probabilities for both a finite and an infinite time horizon, whereas Avram et al. [1] and Avram et al. [2] only considered the infinite time horizon.

The layout of the rest of the article is the following. In this section we present the motivation to study the distribution of the supremum for Lévy processes with a piecewise linear drift and we recall the formulas which will be used in the main results. The next section contains the main results, that is, the distribution of the supremum of a Lévy process with a broken drift and examples. The third section deals with the identities for the Laplace transform of the distribution of the supremum of Lévy processes with a randomly broken drift and on random intervals, which is motivated by a random instant of the premium rate change and a random time when an insurance branch is closed. In the last section we summarize our investigations and discuss future research opportunities.

Let us consider two insurance companies (e.g. insurer and reinsurer) which split the amount they pay out of each claim in proportions $\delta_1 > 0$ and $\delta_2 > 0$ where $\delta_1 + \delta_2 = 1$, and receive premia at rates $p_1 > 0$ and $p_2 > 0$, respectively (see e.g. Avram et al. [2]). Then the corresponding risk processes are

$$R_i(t) = x_i + p_i t - \delta_i X(t),$$

where $i = 1, 2$, $x_i > 0$ and $X(t)$ is the accumulated claim amount up to time t.

One can be interested in the time when at least one insurance company is ruined:

$$\tau_{\text{or}}(x_1, x_2) = \inf\{t > 0 : R_1(t) < 0 \text{ or } R_2(t) < 0\}$$

and in the time when both are ruined:

$$\tau_{\text{sim}}(x_1, x_2) = \inf\{t > 0 : R_1(t) < 0 \text{ and } R_2(t) < 0\}.$$

Let the ultimate ruin probabilities be

$$\psi_{\text{or}}(x_1, x_2) = \mathbb{P}(\tau_{\text{or}}(x_1, x_2) < \infty), \quad \psi_{\text{sim}}(x_1, x_2) = \mathbb{P}(\tau_{\text{sim}}(x_1, x_2) < \infty)$$

and

$$\psi_1(x_1) = \mathbb{P}(\tau_1(x_1) < \infty), \quad \psi_2(x_2) = \mathbb{P}(\tau_2(x_2) < \infty),$$

where $\tau_i(x_i) = \inf\{t > 0 : R_i(t) < 0\}$ for $i = 1, 2$. One can also be interested in the following ruin probability:

$$\psi_{\text{and}}(x_1, x_2) = \mathbb{P}(\tau_1(x_1) < \infty \text{ and } \tau_2(x_2) < \infty),$$

and it is easy to notice that

$$\psi_{\text{and}}(x_1, x_2) = \psi_1(x_1) + \psi_2(x_2) - \psi_{\text{or}}(x_1, x_2).$$

Let us put $u_i = x_i/\delta_i$ and $c_i = p_i/\delta_i$ where $i = 1, 2$. Then we get

$$\tau_{\text{or}}(x_1, x_2) = \inf\{t > 0 : X(t) > u_1 + c_1 t \text{ or } X(t) > u_2 + c_2 t\},$$
$$\tau_{\text{sim}}(x_1, x_2) = \inf\{t > 0 : X(t) > u_1 + c_1 t \text{ and } X(t) > u_2 + c_2 t\}.$$

If the lines $y = u_1 + c_1 t$ and $y = u_2 + c_2 t$ do not cross each other in the first quadrant then the ruin probabilities $\psi_{\text{or}}(x_1, x_2)$ and $\psi_{\text{sim}}(x_1, x_2)$ reduce to ordinary ruin probabilities of a risk process with a linear drift. If they do cross each other in the first quadrant and e.g. $u_1 < u_2$ ($c_1 > c_2$) then

$$\psi_{\text{or}}(x_1, x_2) = \mathbb{P}\Big(\sup_{t<\infty}(X(t) - c(t)) > u_1\Big), \tag{1.1}$$

where $c(t)$ is defined as follows:

$$c(t) = \begin{cases} c_1 t & \text{if } t \in [0, T], \\ c_2(t - T) + c_1 T & \text{if } t \in (T, \infty), \end{cases} \tag{1.2}$$

where $c_1, c_2 \geqslant 0$ and $T = (u_2 - u_1)/(c_1 - c_2)$ (we take $c(t) = \min(u_1 + c_1 t, u_2 + c_2 t) - u_1$).

Similarly, if the lines have a common point in the first quadrant and e.g. $u_2 < u_1$ ($c_2 > c_1$) then

$$\psi_{\text{sim}}(x_1, x_2) = \mathbb{P}\Big(\sup_{t<\infty}(X(t) - c(t)) > u_1\Big), \tag{1.3}$$

where $c(t)$ is defined in (1.2) with $T = (u_2 - u_1)/(c_1 - c_2)$ (we take $c(t) = \max(u_1 + c_1 t, u_2 + c_2 t) - u_1$). In a similar way we can consider ruin probabilities for a finite time horizon. Moreover, the risk process with a broken linear drift, that is, $u + c(t) - X(t)$ $(u > 0)$ can be used to model an insurance portfolio when at some time T the premium rate is changed (e.g. in car insurance because the law regulations have changed). Then the probability in (1.1) or (1.3) describes the ruin probability in this model.

In Michna et al. [20] the joint distribution of the random variable $Y(T)$ and $\inf_{t<T} Y(t)$ was found where Y is a spectrally negative Lévy process (we will consider real stochastic processes with time defined on the non-negative real half-line).

THEOREM 1.1. *If Y is a spectrally negative Lévy process and the one-dimensional distributions of Y are absolutely continuous, then*

$$\mathbb{P}\Big(\inf_{t<T} Y(t) < -u,\, Y(T) + u \in \mathrm{d}z\Big) = \mathrm{d}z \int_0^T \frac{z}{T-s}\, p(z, T-s)\, p(-u, s)\, \mathrm{d}s,$$

where $T, u > 0$, $z \geqslant 0$ and $p(x, s)$ is the density function of $Y(s)$ for $s > 0$.

REMARK 1.1. We do not display a linear drift of the process Y but it can be incorporated in the process Y. Moreover it is not sufficient to assume the absolute continuity of one-dimensional distributions of Y for some $t > 0$ to have it for all $t > 0$ (see Sato [23, Remark 27.22 or Remark 27.24]).

If X is a spectrally positive Lévy process then $X = -Y$ and we get the following corollary.

COROLLARY 1.1. *If X is a spectrally positive Lévy process and the one-dimensional distributions of X are absolutely continuous, then*

$$\mathbb{P}\Big(\sup_{t<T} X(t) \leqslant u,\, X(T) \in \mathrm{d}z\Big) = \Big[f(z, T) - \int_0^T \frac{u-z}{T-s}\, f(z-u, T-s)\, f(u, s)\, \mathrm{d}s\Big]\, \mathrm{d}z,$$

where $T, u > 0$, $z \in (-\infty, u]$ and $f(x, s)$ is the density function of $X(s)$ for $s > 0$.

Integrating the last formula with respect to z we get the following theorem (see Michna et al. [20] and Michna [18]).

THEOREM 1.2. *If the one-dimensional distributions of X are absolutely continuous, then*

$$\mathbb{P}\Big(\sup_{t<T} X(t) > u\Big) = \mathbb{P}(X(T) > u) + \int_0^T \frac{\mathbb{E}(X(T-s))^-}{T-s}\, f(u, s)\, \mathrm{d}s, \tag{1.4}$$

where $x^- = -\min\{x, 0\}$ and $f(u, s)$ is the density function of $X(s)$ for $s > 0$.

REMARK 1.2. The above formula extends the result of Takács [25] to Lévy processes with infinite variation.

Let us now find the joint distribution of the supremum and the value of the process for any spectrally negative Lévy process. It will easily follow from Corollary 1.1 and the duality lemma.

COROLLARY 1.2. *If Y is a spectrally negative Lévy process and the one-dimensional distributions of Y are absolutely continuous, then*

$$\mathbb{P}\Big(\sup_{t<T} Y(t) \leqslant u,\ Y(T) \in \mathrm{d}z\Big) = \Big[p(z,T) - u\int_0^T \frac{p(u,T-s)}{T-s}\, p(z-u,s)\,\mathrm{d}s\Big]\,\mathrm{d}z,$$

where $T, u > 0$, $z \in (-\infty, u]$ and $p(x,s)$ is the density function of $Y(s)$ for $s > 0$.

Proof. By the duality lemma (see e.g. Bertoin [3]) we have $X((T-t)-) - X(T) \stackrel{d}{=} Y(t)$ in the sense of finite-dimensional distributions for $t \leqslant T$ ($X(t-)$ means the left limit at t). Thus we get

$$\begin{aligned}\mathbb{P}\Big(\sup_{t<T} X(t) \leqslant u,\ X(T) \in \mathrm{d}z\Big) &= \mathbb{P}\Big(\sup_{t<T} X((T-t)-) \leqslant u,\ X(T) \in \mathrm{d}z\Big)\\ &= \mathbb{P}\Big(\sup_{t<T}(X((T-t)-) - X(T)) \leqslant u-z,\ X(T) \in \mathrm{d}z\Big)\\ &= \mathbb{P}\Big(\sup_{t<T} Y(t) \leqslant u-z,\ -Y(T) \in \mathrm{d}z\Big).\end{aligned}$$

Substituting $u' = u - z$ and $z' = -z$ and using Corollary 1.1 we obtain the formula. ■

Integrating the last formula with respect to z we could get a result similar to (1.4) for spectrally negative Lévy processes. However we obtain a simpler formula from Kendall's identity (see Kendall [15]). The following theorem can be found in a more general form in Takács [25] (see also Michna [19] for the distribution of supremum for spectrally negative Lévy processes).

THEOREM 1.3. *If Y is a spectrally negative Lévy process and the one-dimensional distributions of Y are absolutely continuous, then*

$$\mathbb{P}\Big(\sup_{t<T} Y(t) > u\Big) = u\int_0^T \frac{p(u,s)}{s}\,\mathrm{d}s,$$

where $p(u,s)$ is the density function of $Y(s)$ for $s > 0$.

Proof. This follows directly from Kendall's identity (see Kendall [15] or e.g. Sato [23, Th. 46.4]). ■

2. MAIN RESULTS AND EXAMPLES

In this section we analyze the distribution of supremum for both $X(t) - c(t)$ and $Y(t) - c(t)$ where X is a spectrally positive Lévy process and Y is a spectrally negative Lévy process and $c(t)$ is defined in (1.2). Since we now display the drift of the process, we will denote the densities of $X(s)$ and $Y(s)$ by $f(x, s)$ and $p(x, s)$, respectively (unlike the previous section where a linear drift could be incorporated in the processes).

THEOREM 2.1. *If $S > T$ (S is finite or $S = \infty$) and $X(t)$ is absolutely continuous with density $f(x, t)$, then*

$$\mathbb{P}\Big(\sup_{t<S}(X(t) - c(t)) > u\Big) = A + B := \mathbb{P}\Big(\sup_{t<T}(X(t) - c_1 t) > u\Big)$$
$$+\mathbb{P}\Big(\sup_{t<T}(X(t)-c_1 t) \leqslant u,\ \sup_{0<t<S-T}\big(X(t+T)-X(T)-c_2 t\big) > u-X(T)+c_1 T\Big),$$

where

$$A = \mathbb{P}(X(T) - c_1 T > u) + \int_0^T \frac{\mathbb{E}(X(T-s) - c_1(T-s))^-}{T-s} f(u + c_1 s, s)\, \mathrm{d}s$$

and

$$B = \int_0^\infty \mathbb{P}\Big(\sup_{t<S-T}(X(t) - c_2 t) > z\Big) f(-z + u + c_1 T, T)\, \mathrm{d}z$$
$$- \int_0^\infty \mathrm{d}z\, z \mathbb{P}\Big(\sup_{t<S-T}(X(t) - c_2 t) > z\Big)$$
$$\cdot \int_0^T \mathrm{d}s\, \frac{f(u + c_1 s, s)}{T-s} f(-z + c_1(T-s), T-s).$$

Proof. The decomposition $A + B$ is obtained as follows:

$$\mathbb{P}\Big(\sup_{t<S}(X(t) - c(t)) > u\Big) = A + B := \mathbb{P}\Big(\sup_{t<T}(X(t) - c_1 t) > u\Big)$$
$$+ \mathbb{P}\Big(\sup_{t<T}(X(t) - c_1 t) \leqslant u,\ \sup_{T<t<S}\big(X(t) - c_2(t-T) - c_1 T\big) > u\Big)$$
$$= \mathbb{P}\Big(\sup_{t<T}(X(t) - c_1 t) > u\Big)$$
$$+ \mathbb{P}\Big(\sup_{t<T}(X(t) - c_1 t) \leqslant u,$$
$$\sup_{0<t<S-T}\big(X(t+T) - X(T) - c_2 t\big) > u - X(T) + c_1 T\Big).$$

The formula for A follows directly from Th. 1.2. Let $F(\mathrm{d}x, \mathrm{d}z)$ be the joint distribution of $(\sup_{t<T}(X(t) - c_1 t), X(T) - c_1 T)$. Then the formula for B follows

from the strong Markov property and Corollary 1.1:

$$\begin{aligned}B &= \int_0^u \int_{-\infty}^u \mathbb{P}\Big(\sup_{t<S-T}(X(t)-c_2t) > u-z\Big)\, F(\mathrm{d}x,\mathrm{d}z)\\ &= \int_{-\infty}^u \mathbb{P}\Big(\sup_{t<S-T}(X(t)-c_2t) > u-z\Big) f(z+c_1T,T)\,\mathrm{d}z\\ &\quad - \int_{-\infty}^u \mathrm{d}z\,\mathbb{P}\Big(\sup_{t<S-T}(X(t)-c_2t) > u-z\Big)\\ &\qquad\qquad \cdot \int_0^T \mathrm{d}s\,\frac{u-z}{T-s} f\big(z-u+c_1(T-s),T-s\big) f(u+c_1s,s)\end{aligned}$$

and substituting $z' = u-z$ we obtain the final formula. ■

Similarly we get a formula for spectrally negative Lévy processes.

THEOREM 2.2. *If $S > T$ (S is finite or $S=\infty$) and $Y(t)$ is absolutely continuous with density $p(x,t)$, then $\mathbb{P}\big(\sup_{t<S}(Y(t)-c(t)) > u\big) = A+B$ where*

$$A = \mathbb{P}\Big(\sup_{t<T}(Y(t)-c_1t) > u\Big) = u\int_0^T \frac{p(u+c_1s,s)}{s}\,\mathrm{d}s$$

and

$$\begin{aligned}B &= \int_0^\infty \mathbb{P}\Big(\sup_{t<S-T}(Y(t)-c_2t) > z\Big) p(-z+u+c_1T,T)\,\mathrm{d}z\\ &\quad - u\int_0^\infty \mathrm{d}z\,\mathbb{P}\Big(\sup_{t<S-T}(Y(t)-c_2t) > z\Big)\\ &\qquad\qquad \cdot \int_0^T \mathrm{d}s\,\frac{p(-z+c_1s,s)}{T-s}\, p(u+c_1(T-s),T-s).\end{aligned}$$

Proof. Using Corollary 1.2 and Th. 1.3 we proceed in the same way as in the proof of Th. 2.1. ■

An application of Th. 2.1 leads to the following example with Brownian motion (see Mandjes [17] and Lieshout and Mandjes [16] or Avram et al. [2]).

EXAMPLE 2.1. If W is the standard Brownian motion, then

$$\begin{aligned}&\mathbb{P}\Big(\sup_{t<\infty}(W(t)-c(t)) > u\Big)\\ &\qquad = \Phi(-uT^{-1/2}-c_1\sqrt{T}) + e^{-2c_1u}\Phi(-uT^{-1/2}+c_1\sqrt{T})\\ &\qquad\quad + e^{-2c_2(u+c_1T-c_2T)}\Phi(uT^{-1/2}+(c_1-2c_2)\sqrt{T})\\ &\qquad\quad - e^{2(c_2-c_1)u+2c_2^2T-2c_1c_2T}\Phi(-uT^{-1/2}+(c_1-2c_2)\sqrt{T}),\end{aligned}$$

where Φ is the cumulative distribution function of the standard normal distribution. Indeed, using Th. 2.1 and

$$\mathbb{P}\Big(\sup_{t<T}(W(t)-ct)>u\Big)=\Phi(-uT^{-1/2}-c\sqrt{T})+e^{-2cu}\Phi(-uT^{-1/2}+c\sqrt{T})$$

and

$$\mathbb{P}\Big(\sup_{t<\infty}(W(t)-ct)>u\Big)=e^{-2cu}$$

for $c\geqslant 0$ (see e.g. Dębicki and Mandjes [11]) we get

$$\mathbb{P}\Big(\sup_{t<\infty}(W(t)-c(t))>u\Big)=A+B, \tag{2.1}$$

where

$$A=A(c_1,T,u):=\Phi(-uT^{-1/2}-c_1\sqrt{T})+e^{-2uc_1}\Phi(-uT^{-1/2}+c_1\sqrt{T}) \tag{2.2}$$

and

$$\begin{aligned}B&=e^{-2c_2(u+c_1T-c_2T)}\Phi(uT^{-1/2}+(c_1-2c_2)\sqrt{T})\\&\quad-\frac{e^{-c_1u-c_1^2T/2}}{2\pi}\int_0^\infty \mathrm{d}z\, ze^{(c_1-2c_2)z}\int_0^T \mathrm{d}s\,(T-s)^{-3/2}s^{-1/2}e^{-\frac{z^2}{2(T-s)}-\frac{u^2}{2s}}.\end{aligned}$$

Let us take $c=c_1=c_2\geqslant 0$ in (2.1). Then $A+B=e^{-2uc}$ and the second summand of A and the first one of B sum to e^{-2uc}, and thus we get

$$\begin{aligned}\frac{e^{-cu-c^2T/2}}{2\pi}\int_0^\infty \mathrm{d}z\, ze^{-cz}\int_0^T \mathrm{d}s\,(T-s)^{-3/2}s^{-1/2}e^{-\frac{z^2}{2(T-s)}-\frac{u^2}{2s}}&\\=\Phi(-uT^{-1/2}-c\sqrt{T}).&\end{aligned}$$

Thus using the last identity for $c=2c_2-c_1\geqslant 0$ we get the second term of B.

Similarly let us take $c=c_1$ and $c_2=0$ in (2.1). Then $A+B=1$ and the first summand of A and the first one of B sum to 1, and thus we get

$$\frac{e^{cu-c^2T/2}}{2\pi}\int_0^\infty \mathrm{d}z\, ze^{cz}\int_0^T \mathrm{d}s\,(T-s)^{-3/2}s^{-1/2}e^{-\frac{z^2}{2(T-s)}-\frac{u^2}{2s}}=\Phi(-uT^{-1/2}+c\sqrt{T}).$$

Thus using the last identity for $c=c_1-2c_2>0$ we get the second term of B.

To demonstrate an application of the above result in insurance let us consider the following example.

EXAMPLE 2.2. Let $X(t)$ be the standard Brownian motion. Then using (1.1) and Example 2.1 we get, for $u_1 < u_2$ and $c_1 > c_2$,

$$\begin{aligned}\psi_{\mathrm{or}}(x_1, x_2) &= \Phi(a(-u_1, -c_1)) + e^{-2c_1 u_1}\Phi(a(-u_1, c_1)) \\ &\quad + e^{-2c_2 u_2}\Phi(a(u_1, c_1 - 2c_2)) - e^{-2(c_1 - 2c_2)u_1 - 2c_2 u_2}\Phi(a(-u_1, c_1 - 2c_2)),\end{aligned}$$

where $a(u, c) = uT^{-1/2} + c\sqrt{T}$, $T = (u_2 - u_1)/(c_1 - c_2)$, $u_i = x_i/\delta_i$ and $c_i = p_i/\delta_i$ for $i = 1, 2$. This formula recovers the result of Avram et al. [2, (67)]. In the same way we obtain a formula for $\psi_{\mathrm{sim}}(x_1, x_2)$.

EXAMPLE 2.3. Let $0 < T < S < \infty$ and let W be the standard Brownian motion. Then

$$\begin{aligned}&\mathbb{P}\Big(\sup_{t<S}(W(t) - c(t)) > u\Big) \\ &= A(c_1, T, u) + \frac{1}{\sqrt{2\pi T}}\int_0^\infty A(c_2, S - T, z)e^{-\frac{(u+c_1T-z)^2}{2T}}\,\mathrm{d}z \\ &\quad - \frac{e^{-uc_1 - \frac{c_1^2 T}{2}}}{2\pi}\int_0^\infty \mathrm{d}z\, z e^{c_1 z}A(c_2, S - T, z)\int_0^T \mathrm{d}s\, s^{-1/2}(T - s)^{-3/2}e^{-\frac{z^2}{2(T-s)} - \frac{u^2}{2s}},\end{aligned}$$

where $A(c_1, T, u)$ is defined in (2.2).

EXAMPLE 2.4. Let $X(t)$ be the gamma Lévy process with density

$$f(x, t) = \frac{\delta^t}{\Gamma(t)}x^{t-1}e^{-\delta x}\mathbb{1}(x > 0),$$

where $\delta > 0$, and let $c(t)$ be defined in (1.2). Using Th. 2.1 we give explicit formulas for $\mathbb{P}(\sup_{t<S}(X(t) - c(t)) > u) = A + B$ for both $T < S < \infty$ and $S = \infty$. For $T < S < \infty$, we have

$$\begin{aligned}A &= \frac{\delta^T}{\Gamma(T)}\int_{u+c_1T}^\infty x^{T-1}e^{-\delta x}\,\mathrm{d}x \\ &\quad + \delta^T e^{-\delta u}\int_0^T \mathrm{d}s\,\frac{(u + c_1 s)^{s-1}e^{-c_1\delta s}}{\Gamma(s)\Gamma(T - s + 1)}\int_0^{c_1(T-s)} \mathrm{d}x\,(c_1(T - s) - x)x^{T-s-1}e^{-\delta x} \\ &=: A(c_1, T, u)\end{aligned}$$

and

$$B = \frac{\delta^S e^{-\delta(u+c_1T)}}{\Gamma(T)\Gamma(S-T)} \int_0^{u+c_1T} \mathrm{d}z\,(u+c_1T-z)^{T-1} e^{\delta z} \int_{z+c_2(S-T)}^{\infty} \mathrm{d}x\, x^{S-T-1} e^{-\delta x}$$
$$+ \frac{\delta^S e^{-\delta(u+c_1T)}}{\Gamma(T)} \int_0^{u+c_1T} \mathrm{d}z\,(u+c_1T-z)^{T-1} \int_0^{S-T} \mathrm{d}s\, \frac{(z+c_2s)^{s-1}e^{-c_2\delta s}}{\Gamma(s)\Gamma(S-T-s+1)}$$
$$\cdot \int_0^{c_2(S-T-s)} \mathrm{d}x\,(c_2(S-T-s)-x)x^{S-T-s-1}e^{-\delta x}$$
$$- \delta^T e^{-\delta(u+c_1T)} \int_0^{c_1T} \mathrm{d}z\, z e^{\delta z} A(c_2, S-T, z)$$
$$\cdot \int_0^{\frac{c_1T-z}{c_1}} \mathrm{d}s\, \frac{(u+c_1s)^{s-1}(c_1(T-s)-z)^{T-s-1}}{\Gamma(s)\Gamma(T-s+1)}.$$

For $S=\infty$, we additionally assume that $c_2\delta > 1$. In this case, since $X(t)$ has finite variation, in view of Th. 4 in Takács [25] we have

$$\mathbb{P}\Big(\sup_{t<\infty}(X(t)-c_2t) > z\Big) = \frac{c_2\delta-1}{\delta}\, e^{-\delta z} \int_0^{\infty} \frac{\delta^s}{\Gamma(s)} (z+c_2s)^{s-1} e^{-\delta c_2 s}\,\mathrm{d}s, \qquad z>0.$$

Let us notice that A is the same as in the case $T<S<\infty$ and using the above expression we get

$$B = \frac{(c_2\delta-1)\delta^{T-1}e^{-\delta(u+c_1T)}}{\Gamma(T)} \int_0^{u+c_1T} \mathrm{d}z\,(u+c_1T-z)^{T-1}$$
$$\cdot \int_0^{\infty} \mathrm{d}s\, \frac{\delta^s}{\Gamma(s)} (z+c_2s)^{s-1} e^{-\delta c_2 s}$$
$$- (c_2\delta-1)\delta^{T-1}e^{-\delta(u+c_1T)} \int_0^{c_1T} \mathrm{d}z\, z$$
$$\cdot \int_0^{T-z/c_1} \mathrm{d}s\, \frac{(u+c_1s)^{s-1}(c_1(T-s)-z)^{T-s-1}}{\Gamma(s)\Gamma(T-s+1)} \int_0^{\infty} \mathrm{d}t\, \frac{\delta^t}{\Gamma(t)} (z+c_2t)^{t-1} e^{-\delta c_2 t}.$$

EXAMPLE 2.5. Let $Z(s)$ be an α-stable Lévy process totally skewed to the right (that is, with $\beta=1$; see e.g. Janicki and Weron [14] or Samorodnitsky and Taqqu [22]) with $1<\alpha<2$ and expectation zero. Its density function is

$$f(x,s) = \frac{1}{\pi s^{1/\alpha}} \int_0^{\infty} e^{-t^\alpha} \cos\Big(t s^{-1/\alpha} x - t^\alpha \tan\frac{\pi\alpha}{2}\Big)\,\mathrm{d}t$$

(see e.g. Nolan [21]). Then (see Michna et al. [20]), for $c > 0$,

$$A(c,\infty,u) := \mathbb{P}\Big(\sup_{t<\infty}(Z(t)-ct) > u\Big)$$
$$= \frac{c}{\pi}\int_0^\infty \mathrm{d}s\, s^{-1/\alpha}\int_0^\infty \mathrm{d}t\, e^{-t^\alpha}\cos\Big(ts^{-1/\alpha}(u+cs) - t^\alpha\tan\frac{\pi\alpha}{2}\Big),$$

and for any c and $T > 0$,

$$A(c,T,u) := \mathbb{P}\Big(\sup_{t<T}(Z(t)-ct) > u\Big) \tag{2.3}$$
$$= \frac{1}{\pi T^{1/\alpha}}\int_u^\infty \mathrm{d}x\int_0^\infty \mathrm{d}t\, e^{-t^\alpha}\cos\Big(tT^{-1/\alpha}(x+cT) - t^\alpha\tan\frac{\pi\alpha}{2}\Big)$$
$$+ \frac{1}{\pi}\int_0^T \mathrm{d}s\,\frac{\mathbb{E}(Z(T-s)-c(T-s))^-}{(T-s)s^{1/\alpha}}$$
$$\cdot\int_0^\infty \mathrm{d}t\, e^{-t^\alpha}\cos\Big(ts^{-1/\alpha}(u+cs) - t^\alpha\tan\frac{\pi\alpha}{2}\Big),$$

where

$$\mathbb{E}(Z(s)-cs)^- = \frac{-1}{\pi s^{1/\alpha}}\int_{-\infty}^0 \mathrm{d}x\, x\int_0^\infty \mathrm{d}t\, e^{-t^\alpha}\cos\Big(ts^{-1/\alpha}(x+cs) - t^\alpha\tan\frac{\pi\alpha}{2}\Big).$$

Thus using Th. 2.1 for $S > T > 0$ (allowing also $S = \infty$ and putting $\infty - T = \infty$) we get

$$\mathbb{P}\Big(\sup_{t<S}(Z(t)-c(t)) > u\Big) = A + B = A + B_1 - B_2,$$

where $A = A(c_1,T,u)$ (see (2.3)) and

$$B_1 = \frac{1}{\pi T^{1/\alpha}}\int_0^\infty \mathrm{d}z\, A(c_2, S-T, z)$$
$$\cdot\int_0^\infty \mathrm{d}t\, e^{-t^\alpha}\cos\Big(tT^{-1/\alpha}(-z+u+c_1T) - t^\alpha\tan\frac{\pi\alpha}{2}\Big)$$

and

$$B_2 = \frac{1}{\pi^2}\int_0^\infty \mathrm{d}z\, zA(c_2,S-T,z)\int_0^T \frac{\mathrm{d}s}{(T-s)^{1/\alpha+1}s^{1/\alpha}}$$
$$\cdot\int_0^\infty \mathrm{d}t\, e^{-t^\alpha}\cos\Big(ts^{-1/\alpha}(u+c_1s) - t^\alpha\tan\frac{\pi\alpha}{2}\Big)$$
$$\cdot\int_0^\infty \mathrm{d}w\, e^{-w^\alpha}\cos\Big(w(T-s)^{-1/\alpha}(-z+c_1(T-s)) - w^\alpha\tan\frac{\pi\alpha}{2}\Big).$$

3. RANDOMLY BROKEN DRIFT AND RANDOM INTERVAL

In fluctuation theory there are many interesting identities for Lévy processes and exponentially distributed time, e.g. the distribution of supremum on an exponentially distributed time interval (see e.g. Bertoin [3, Secs. VI.2 and VII]). Let us consider a spectrally positive Lévy process X with a randomly broken drift, that is, assume that T (see (1.2)) is an exponentially distributed random variable with mean $1/\lambda$ independent of the process X. Moreover let us investigate two cases: when $S=\infty$ (see Th. 2.1) and when $S-T=V$ is a positive random variable independent of the process X and the random variable T. The practical motivation for this model is the above mentioned risk process with a changing premium rate at time T; we can think that at a random time T the premium rate is changed regardless of claims (e.g. the law regulations are changed in car insurance). Moreover a finite random time horizon S can be viewed as the random time at which the insurance company suspends a certain insurance portfolio (e.g. creates a new portfolio) and is interested in the probability of ruin before time S. For this model we give the Laplace transforms of the survival probability.

Let us put

$$\varphi_i(\gamma)=\ln\mathbb{E}\exp(-\gamma(X(1)-c_i)),\qquad i=1,2,$$

where $\gamma\geqslant 0$ and let $\overleftarrow{\varphi}_i(\lambda)$, $i=1,2$, be the inverse function of φ_i.

THEOREM 3.1. *If X is a spectrally positive Lévy process and T is an exponential random variable with mean $1/\lambda>0$ independent of X, then for any $\gamma>\overleftarrow{\varphi}_1(\lambda)$,*

$$\begin{aligned}(3.1)\qquad \mathbb{E}e^{-\gamma\sup_{t<T+V}(X(t)-c(t))}&=\mathbb{E}e^{-\gamma\sup_{t<T}(X(t)-c_1t)}\\ &+\frac{\gamma\lambda}{\varphi_1(\gamma)-\lambda}\left[\frac{1-\mathbb{E}e^{-\gamma\sup_{t<V}(X(t)-c_2t)}}{\gamma}-\frac{1-\mathbb{E}e^{-\overleftarrow{\varphi}_1(\lambda)\sup_{t<V}(X(t)-c_2t)}}{\overleftarrow{\varphi}_1(\lambda)}\right]\end{aligned}$$

where V is a positive random variable independent of X and T.

Proof. Observe that for $\gamma>0$,

$$\mathbb{E}e^{-\gamma\sup_{t<T+V}(X(t)-c(t))}=1-\gamma\int_0^\infty e^{-\gamma u}\mathbb{P}\Big(\sup_{t<T+V}(X(t)-c(t))>u\Big)\,\mathrm{d}u$$

and

$$\begin{aligned}\mathbb{P}\Big(\sup_{t<T+V}(X(t)-c(t))>u\Big)&=\mathbb{P}\Big(\sup_{t<T}(X(t)-c_1t)>u\Big)\\ &+\mathbb{P}\Big(\sup_{t<T}(X(t)-c_1t)\leqslant u,\ \sup_{t<V}(X(t+T)-X(T)-c_2t)>u-X(T)+c_1T\Big).\end{aligned}$$

Thus

$$\int_0^\infty e^{-\gamma u}\mathbb{P}\Big(\sup_{t<T+V}(X(t)-c(t))>u\Big)\,\mathrm{d}u = I_1 + I_2, \tag{3.2}$$

where

$$I_1 := \int_0^\infty e^{-\gamma u}\mathbb{P}\Big(\sup_{t<T}(X(t)-c_1t)>u\Big)\,\mathrm{d}u = \frac{1-\mathbb{E}e^{-\gamma\sup_{t<T}(X(t)-c_1t)}}{\gamma}$$

and

$$I_2 := \int_0^\infty e^{-\gamma u}\mathbb{P}\Big(\sup_{t<T}(X(t)-c_1t)\leqslant u,\\ \sup_{t<V}\big(X(t+T)-X(T)-c_2t\big)>u-X(T)+c_1T\Big)\,\mathrm{d}u.$$

Since T is exponentially distributed and independent of X and V, we have

$$I_2 = \lambda\int_0^\infty \mathrm{d}s\, e^{-\lambda s}\int_0^\infty \mathrm{d}u\, e^{-\gamma u}\\ \cdot\mathbb{P}\Big(\sup_{t<s}(X(t)-c_1t)\leqslant u,\, \sup_{t<V}\big(X(t+s)-X(s)-c_2t\big)>u-X(s)+c_1s\Big).$$

Moreover, by the independence of $X(t+s)-X(s)-c_2t$, $t\geqslant 0$, and $X(s)-c_1s$ and the fact that

$$\mathbb{P}\Big(\sup_{t<s}(X(t)-c_1t)\leqslant u,\, u-X(s)+c_1s\leqslant z\Big)=0,\qquad z<0,$$

we have

$$I_2 = \lambda\int_0^\infty \mathrm{d}s\, e^{-\lambda s}\int_0^\infty \mathrm{d}u\, e^{-\gamma u}\int_0^\infty \mathbb{P}\Big(\sup_{t<V}(X(t)-c_2t)>z\Big)\\ \cdot\mathbb{P}\Big(\sup_{t<s}(X(t)-c_1t)\leqslant u,\, u-X(s)+c_1s\in\,\mathrm{d}z\Big)\\ = \lambda\int_0^\infty \mathrm{d}u\, e^{-\gamma u}\int_0^\infty \mathrm{d}s\, e^{-\lambda s}\int_0^\infty \mathbb{P}\Big(\sup_{t<V}(X(t)-c_2t)>z\Big)\\ \cdot\mathbb{P}\Big(\inf_{t<s}(u-X(t)+c_1t)>0,\, u-X(s)+c_1s\in\,\mathrm{d}z\Big).$$

Due to Suprun [24] (see also Bertoin [4, Lemma 1]) we have

$$\int_0^\infty e^{-\lambda s}\mathbb{P}\Big(\inf_{t<s}(u-X(t)+c_1t)>0,\, u-X(s)+c_1s\in\,\mathrm{d}z\Big)\,\mathrm{d}s\\ = \big[e^{-\overleftarrow{\varphi}_1(\lambda)z}W^{(\lambda)}(u)-\mathbb{1}(u\geqslant z)W^{(\lambda)}(u-z)\big]\,\mathrm{d}z,$$

where $\mathbb{1}(\cdot)$ is the indicator function and $W^{(\lambda)} : [0,\infty) \to [0,\infty)$ is a continuous and increasing function such that

$$\int_0^\infty e^{-\gamma x}\, W^{(\lambda)}(x)\,\mathrm{d}x = \frac{1}{\varphi_1(\gamma)-\lambda}, \qquad \gamma > \overleftarrow{\varphi}_1(\lambda).$$

Consequently, for $\gamma > \overleftarrow{\varphi}_1(\lambda)$,

$$\begin{aligned}
I_2 &= \lambda \int_0^\infty \int_0^\infty e^{-\gamma u}\mathbb{P}\Big(\sup_{t<V}(X(t)-c_2t) > z\Big) \\
&\qquad\qquad \cdot [e^{-\overleftarrow{\varphi}_1(\lambda)z} W^{(\lambda)}(u) - \mathbb{1}(u \geqslant z) W^{(\lambda)}(u-z)]\,\mathrm{d}z\,\mathrm{d}u \\
&= \lambda \int_0^\infty \mathrm{d}z\, e^{-\overleftarrow{\varphi}_1(\lambda)z}\mathbb{P}\Big(\sup_{t<V}(X(t)-c_2t) > z\Big) \int_0^\infty \mathrm{d}u\, e^{-\gamma u} W^{(\lambda)}(u) \\
&\quad - \lambda \int_0^\infty \mathrm{d}z\, \mathbb{P}\Big(\sup_{t<V}(X(t)-c_2t) > z\Big) \int_0^\infty \mathrm{d}u\, \mathbb{1}(u \geqslant z) e^{-\gamma u} W^{(\lambda)}(u-z) \\
&= \frac{\lambda}{\varphi_1(\gamma)-\lambda}\Big[\int_0^\infty e^{-\overleftarrow{\varphi}_1(\lambda)z}\mathbb{P}\Big(\sup_{t<V}(X(t)-c_2t) > z\Big)\,\mathrm{d}z \\
&\quad - \int_0^\infty e^{-\gamma z}\mathbb{P}\Big(\sup_{t<V}(X(t)-c_2t) > z\Big)\,\mathrm{d}z\Big] \\
&= \frac{\lambda}{\varphi_1(\gamma)-\lambda}\left[\frac{1-\mathbb{E}e^{-\overleftarrow{\varphi}_1(\lambda)\sup_{t<V}(X(t)-c_2t)}}{\overleftarrow{\varphi}_1(\lambda)} - \frac{1-\mathbb{E}e^{-\gamma \sup_{t<V}(X(t)-c_2t)}}{\gamma}\right]. \quad \blacksquare
\end{aligned}$$

COROLLARY 3.1. *Under the assumption of Theorem* 3.1, *if* $V=\infty$, *then*

$$\mathbb{E}e^{-\gamma \sup_{t<\infty}(X(t)-c(t))} = \frac{\gamma\lambda\varphi_2'(0)[\varphi_2(\gamma)-\varphi_2(\overleftarrow{\varphi}_1(\lambda))]}{\varphi_2(\gamma)(\varphi_1(\gamma)-\lambda)\varphi_2(\overleftarrow{\varphi}_1(\lambda))}. \tag{3.3}$$

If V *is an exponentially distributed random variable with mean* $1/\theta > 0$ *independent of* X *and* T *then*

$$\mathbb{E}e^{-\gamma \sup_{t<T+V}(X(t)-c(t))} = \gamma\lambda\theta\, \frac{\frac{\overleftarrow{\varphi}_2(\theta)-\overleftarrow{\varphi}_1(\lambda)}{\theta-\varphi_2(\overleftarrow{\varphi}_1(\lambda))} - \frac{\overleftarrow{\varphi}_1(\lambda)[\overleftarrow{\varphi}_2(\theta)-\gamma]}{\gamma[\theta-\varphi_2(\gamma)]}}{\overleftarrow{\varphi}_1(\lambda)\overleftarrow{\varphi}_2(\theta)[\varphi_1(\gamma)-\lambda]}. \tag{3.4}$$

Proof. Case $V=\infty$. It is well-known that

$$\mathbb{E}e^{-\gamma \sup_{t<T}(X(t)-c_1t)} = \frac{\lambda}{\lambda-\varphi_1(\gamma)}\Big(1-\frac{\gamma}{\overleftarrow{\varphi}_1(\lambda)}\Big), \tag{3.5}$$

where $\gamma \geqslant 0$ (see e.g. Bertoin [3, (3) p. 192] or Dębicki and Mandjes [11, Th. 4.1]). Moreover, by [11, Th. 3.2] (or letting $\lambda \to 0$ in (3.5)), it follows that

$$\mathbb{E}\exp\Big(-\gamma \sup_{t<\infty}(X(t)-c_2t)\Big) = \frac{\gamma\varphi_2'(0)}{\varphi_2(\gamma)}.$$

Consequently, by (3.1) for $\gamma > 0$,

$$\mathbb{E}e^{-\gamma \sup_{t<\infty}(X(t)-c(t))}$$

$$= \frac{\lambda}{\lambda - \varphi_1(\gamma)}\left[1 - \frac{\gamma}{\overleftarrow{\varphi}_1(\lambda)}\right] + \frac{\gamma\lambda}{\varphi_1(\gamma)-\lambda}\left[\frac{1 - \frac{\gamma\varphi_2'(0)}{\varphi_2(\gamma)}}{\gamma} - \frac{1 - \frac{\overleftarrow{\varphi}_1(\lambda)\varphi_2'(0)}{\varphi_2(\overleftarrow{\varphi}_1(\lambda))}}{\overleftarrow{\varphi}_1(\lambda)}\right]$$
$$= \frac{\gamma\lambda\varphi_2'(0)[\varphi_2(\gamma) - \varphi_2(\overleftarrow{\varphi}_1(\lambda))]}{\varphi_2(\gamma)(\varphi_1(\gamma)-\lambda)\varphi_2(\overleftarrow{\varphi}_1(\lambda))}.$$

Case of V exponentially distributed. Using (3.5), for $\gamma \geqslant 0$ we have

$$\mathbb{E}\exp\Big(-\gamma \sup_{t<V}(X(t) - c_2 t)\Big) = \frac{\theta}{\theta - \varphi_2(\gamma)}\Big(1 - \frac{\gamma}{\overleftarrow{\varphi}_2(\theta)}\Big).$$

Recalling (3.5), for $\gamma > 0$ it follows that

$$\mathbb{E}e^{-\gamma \sup_{t<T+V}(X(t)-c(t))}$$
$$= \frac{\lambda}{\lambda - \varphi_1(\gamma)}\Big(1 - \frac{\gamma}{\overleftarrow{\varphi}_1(\lambda)}\Big)$$
$$+ \frac{\gamma\lambda}{\varphi_1(\gamma) - \lambda}\left[\frac{1 - \frac{\theta}{\theta-\varphi_2(\gamma)}\big[1 - \frac{\gamma}{\overleftarrow{\varphi}_2(\theta)}\big]}{\gamma} - \frac{1 - \frac{\theta}{\theta - \varphi_2(\overleftarrow{\varphi}_1(\lambda))}\big[1 - \frac{\overleftarrow{\varphi}_1(\lambda)}{\overleftarrow{\varphi}_2(\theta)}\big]}{\overleftarrow{\varphi}_1(\lambda)}\right]$$
$$= \gamma\lambda\theta\,\frac{\frac{\overleftarrow{\varphi}_2(\theta) - \overleftarrow{\varphi}_1(\lambda)}{\theta - \varphi_2(\overleftarrow{\varphi}_1(\lambda))} - \frac{\overleftarrow{\varphi}_1(\lambda)[\overleftarrow{\varphi}_2(\theta)-\gamma]}{\gamma[\theta-\varphi_2(\gamma)]}}{\overleftarrow{\varphi}_1(\lambda)\overleftarrow{\varphi}_2(\theta)[\varphi_1(\gamma)-\lambda]}.$$ ■

COROLLARY 3.2. *Let W be the standard Brownian motion. Then*

$$\varphi_1(\gamma) = \tfrac{1}{2}\gamma^2 + c_1\gamma, \qquad \varphi_2(\gamma) = \tfrac{1}{2}\gamma^2 + c_2\gamma,$$
$$\overleftarrow{\varphi}_1(\lambda) = \sqrt{c_1^2 + 2\lambda} - c_1, \quad \overleftarrow{\varphi}_2(\lambda) = \sqrt{c_2^2 + 2\lambda} - c_2.$$

Thus for $\gamma > \sqrt{c_1^2 + 2\lambda} - c_1$,

$$\mathbb{E}e^{-\gamma \sup_{t<\infty}(W(t)-c(t))}$$
$$= \frac{\gamma\lambda c_2(\frac{1}{2}\gamma^2 + c_2\gamma - c_1^2 - \lambda - (c_2 - c_1)\sqrt{c_1^2+2\lambda} + c_1c_2)}{(\frac{1}{2}\gamma^2 + c_1\gamma - \lambda)(\frac{1}{2}\gamma^2 + c_2\gamma)(c_1^2 + \lambda + (c_2-c_1)\sqrt{c_1^2+2\lambda} - c_1c_2)}$$

and

$$\mathbb{E}e^{-\gamma \sup_{t<T+V}(W(t)-c(t))}$$
$$= \gamma\lambda\theta\,\frac{\frac{\sqrt{c_2^2+2\theta} - \sqrt{c_1^2+2\lambda} + c_1 - c_2}{\theta - c_1^2 - \lambda - (c_2-c_1)\sqrt{c_1^2+2\lambda} + c_1c_2} - \frac{(\sqrt{c_1^2+2\lambda} - c_1)(\sqrt{c_2^2+2\theta} - c_2 - \gamma)}{\gamma(\theta - \frac{1}{2}\gamma^2 - c_2\gamma)}}{(\sqrt{c_1^2+2\lambda} - c_1)(\sqrt{c_2^2+2\theta} - c_2)(\frac{1}{2}\gamma^2 + c_1\gamma - \lambda)}.$$

4. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

We have considered the ruin time in the classical way when two insurance companies share the same claims and premia in a proportional way or when the premium rate is changed at a certain time for a given insurance branch. For these models we have derived exact formulas for the ruin probability for a finite and an infinite time horizon when the accumulated claim amount process is described by a spectrally asymmetric Lévy process. Since the accumulated claim amount process is the same for both insurance companies in our first model, a generalization which comes to mind is to analyze two risk processes where the accumulated claim amount processes are not the same but they are correlated (if possible) with correlation coefficient $-1 < \rho < 1$ (see Dębicki et al. [10] in the case of correlated Brownian motions). Moreover there exist other types of ruin which attract practitioners' attention and can be applied in modeling two collaborating insurance companies or the change of the premium rate for a given insurance portfolio. The Parisian type of ruin has now been deeply investigated. The Parisian ruin occurs if the risk process stays below zero longer than a fixed period of time. This concept was introduced by Dassios and Wu [8] and received a great deal of attention from probabilists and practitioners (see e.g. Dębicki et al. [9] or Czarna and Palmowski [7] and the references therein). This type of ruin can be extended to the Parisian ruin with a lower ultimate bankrupt barrier, where the ruin occurs if the process stays below zero longer than a fixed amount of time or goes below a fixed level $-a$ ($a > 0$ sufficiently large, see e.g. Czarna [6] and the references therein). Thus these types of ruin can be implemented in models where two insurance companies collaborate by sharing claims and premia or when the premium rate is changed in a certain insurance portfolio. In the above mentioned articles exact formulas or an asymptotic behavior for the ruin probability are determined, but there are other ways to approximate the ruin probability: see e.g. Burnecki et al. [5] where De Vylder type approximation of the ruin probability for two collaborating insurance companies is introduced. This idea could also be implemented for other types of ruin in the insurer-reinsurer model.

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Zbigniew Michna
Department of Logistics
Wrocław University of Economics and Business
53-345 Wrocław, Poland
E-mail: zbigniew.michna@ue.wroc.pl

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