

ON THE EXACT ASYMPTOTICS OF EXIT TIME FROM A CONE OF AN
ISOTROPIC α -SELF-SIMILAR MARKOV PROCESS WITH A
SKEW-PRODUCT STRUCTURE*

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Abstract. In this paper we identify the asymptotic tail of the distribution of the exit time τ_C from a cone C of an isotropic α -self-similar Markov process X_t with a skew-product structure, that is, X_t is a product of its radial process and an independent time changed angular component Θ_t . Under some additional regularity assumptions, the angular process Θ_t killed on exiting the cone C has a transition density that can be expressed in terms of a complete set of orthogonal eigenfunctions with corresponding eigenvalues of an appropriate generator. Using this fact and some asymptotic properties of the exponential functional of a killed Lévy process related to the Lamperti representation of the radial process, we prove that

$$\mathbb{P}_x(\tau_C > t) \sim h(x)t^{-\kappa_1}$$

as $t \rightarrow \infty$ for h and κ_1 identified explicitly. The result extends the work of De Blassie (1988) and Bañuelos and Smits (1997) concerning the Brownian motion.

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Key words and phrases: α -self-similar process, cone, exit time, skew-product structure, Lamperti representation, exponential functional, Brownian motion.

THE FULL TEXT IS AVAILABLE HERE

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