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EXTREMES OF MULTIDIMENSIONAL STATIONARY GAUSSIAN RANDOM FIELDS

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Abstract: Let $\{X(\mathbf{t}) : \mathbf{t} = (t_1, t_2, \ldots, t_d) \in [0,\infty)^d\}$ be a centered stationary Gaussian field with almost surely continuous sample paths, unit variance and correlation function r satisfying $r(\mathbf{t}) < 1$ for every $\mathbf{t} \neq \mathbf{0}$ and $r(\mathbf{t}) = 1 - \sum_{i=1}^d |t_i|^{\alpha_i} + o\big(\sum_{i=1}^d |t_i|^{\alpha_i}\big)$, as $\mathbf{t} \to \mathbf{0}$, with some $\alpha_1, \alpha_2, \ldots, \alpha_d \in (0,2]$. The main result of this contribution is the description of the asymptotic behaviour of $\mathrm{P}\left(\sup\{X(\mathbf{t}) : \mathbf{t} \in \mathcal{J}_{\mathbf{m}}^{\mathbf{x}}\} \leqslant u\right)$, as $u \to \infty$, for some Jordan-measurable sets $\mathcal{J}_{\mathbf{m}}^{\mathbf{x}}$ of volume proportional to $\mathrm{P}\left(\sup\{X(\mathbf{t}) : \mathbf{t} \in [0,1]^d\} > u\right)^{-1}\big(1 + o(1)\big)$.

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