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# A TWO-PARAMETER EXTENSION OF URBANIK'S PRODUCT CONVOLUTION SEMIGROUP

**Christian Berg**

*Abstract:* We prove that $s_n(a,b) = \Gamma(an+b)/\Gamma(b), n = 0,1,\ldots$, is an infinitely divisible Stieltjes moment sequence for arbitrary $a, b > 0$. Its powers $s_n(a,b)^c, c > 0$, are Stieltjes determinate if and only if $ac \leq 2$. The latter was conjectured in a paper by Lin (2019) in the case $b = 1$. We describe a product convolution semigroup $\tau_c(a,b)$, $c > 0$, of probability measures on the positive half-line with densities $e_c(a,b)$ and having the moments $s_n(a,b)^c$. We determine the asymptotic behavior of $e_c(a,b)(t)$ for $t \to 0$ and for $t \to \infty$, and the latter implies the Stieltjes indeterminacy when $ac > 2$. The results extend the previous work of the author and López (2015) and lead to a convolution semigroup of probability densities $\big(g_c(a,b)(x)\big)_{c>0}$ on the real line. The special case $\big(g_c(a,1)(x)\big)_{c>0}$ are the convolution roots of the Gumbel distribution with scale parameter $a > 0$. All the densities $g_c(a,b)(x)$ lead to determinate Hamburger moment problems.

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