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LIMIT BEHAVIOR OF THE INVARIANT MEASURE FOR LANGEVIN DYNAMICS

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Abstract. We consider the Langevin dynamics on $\mathbb{R}^d$ with an overdamped vector field and driven by multiplicative Brownian noise of small amplitude $\sqrt{\epsilon}$, $\epsilon > 0$. Under suitable assumptions on the vector field and the diffusion coefficient, it is well-known that it has a unique invariant probability measure μ^ϵ. We prove that as ϵ tends to zero, the probability measure $\epsilon^{d/2}\mu^\epsilon(\sqrt{\epsilon}\,\mathrm{d}x)$ converges in the p-Wasserstein distance for $p \in [1,2]$ to a Gaussian measure with zero-mean vector and non-degenerate covariance matrix which solves a Lyapunov matrix equation. Moreover, the error term is estimated. We emphasize that generically no explicit formula for μ^ϵ can be found.

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