Algebraic Topology 2. Exercises.
List 3.

Local degree

1. For any $x \in S^n$ the group $H_n(S^n, S^n \setminus \{x\})$ can be naturally identified with the group $H_n S^n$, via the homomorphism $j_* : H_n S^n \to H_n(S^n, S^n \setminus \{x\})$ in the long exact sequence of the pair $(S^n, S^n \setminus \{x\})$. This allows to define the *local degree* at a point $x \in U$ for any homeomorphism $h : U \to V$ between open subsets of S^n, by using excision.
   (a) Show that for such local degree we always have $\deg(h|x) = \pm 1$.
   (b) Show that if $r : S^n \to S^n$ is any reflection with respect to some equatorial $S^{n-1} \subset S^n$ then $\deg(r \circ h|x) = \deg(h \circ r|r(x)) = -\deg(h|x)$.
   (c) Show that the local degree $\deg(h|x)$ does not depend on the choice of $x \in U$.

Computations of cellular homology

Exercises 17 and 28-29 from page 132, and exercises 9, 10, 12, 14, 19 from page 156 of Hatcher's book "Algebraic Topology".