Mamy

$$\begin{aligned}\|(L_a+\lambda I)b\|^2 = \|ab+\lambda b\|^2 = \|(ab+\lambda b)^*(ab+\lambda b)\| = \|b^*(L_a+\lambda I)^*(L_a+\lambda I)b\| \\ \leqslant \|b\|\,\|(L_a+\lambda I)^*(L_a+\lambda I)b\| \leqslant \|b\|^2\|(L_a+\lambda I)^*(L_a+\lambda I)\|\end{aligned}$$

Zatem

$$\|L_a+\lambda I\|^2 \leqslant \|(L_a+\lambda I)^*(L_a+\lambda I)\|.$$