Rozważamy funkcjonały

$$\varphi_n(f) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}} f(x)\,dx, \quad f \in C[-\pi,\pi].$$

Oznaczmy

$$f_n(x) = \left|\sin\left(n+\frac{1}{2}\right)x\right|.$$

Wtedy $\|f_n\|_\infty = 1$ oraz

$$\varphi_n(f_n) = \frac{1}{\pi}\int_0^{\pi} \frac{\sin^2\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}\,dx = \frac{2}{\pi}\int_0^{\pi/2} \frac{\sin^2(2n+1)x}{\sin x}\,dx$$

$$\geqslant \frac{2}{\pi}\int_0^{\pi/2} \frac{\sin^2(2n+1)x}{x}\,dx = \frac{2}{\pi}\int_0^{\left(n+\frac{1}{2}\right)\pi} \frac{\sin^2 x}{x}\,dx \geqslant \frac{1}{\pi}\int_{\frac{\pi}{2}}^{\left(n+\frac{1}{2}\right)\pi} \frac{1-\cos 2x}{x}\,dx$$

$$= \frac{1}{\pi}\log(2n+1) - \frac{1}{\pi}\int_{\pi}^{(2n+1)\pi} \frac{\cos x}{x}\,dx \geqslant \frac{1}{\pi}\log(2n+1).$$

Ostatnia nierówność wynika z

$$\int_{\pi}^{(2n+1)\pi} \frac{\cos x}{x}\,dx = \int_{\pi}^{(2n+1)\pi} \frac{\sin x}{x^2}\,dx = \int_0^{\pi} \sum_{k=1}^{2n} \frac{(-1)^k}{(x+k\pi)^2}\ \sin x\,dx$$

oraz

$$\sum_{k=1}^{2n} \frac{(-1)^{k+1}}{(x+k\pi)^2} = \sum_{k=1}^{n} \left\{ \frac{1}{[x+(2k-1)x]^2} - \frac{1}{(x+2k\pi)^2} \right\} > 0.$$

Reasumując

$$\|\varphi_n\|_{C[-\pi,\pi]^*} \geqslant \frac{1}{\pi}\log(2n+1).$$