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ALMOST SURE CONVERGENCE OF THE DISTRIBUTIONAL LIMIT THEOREM FOR ORDER STATISTICS

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Abstract: Let $X_n, n \geq 1$, be a sequence of independent and identically distributed random variables and $X_{n,1} \leq X_{n,2} \leq \ldots \leq X_{n,n}$ denote the order statistics of $X_1, \ldots, X_n$. For any sequence of integers $\{k_n\}$ with $1 \leq k_n \leq n$ and $\lim_{n\to\infty} \min\{k_n, n - k_n + 1\} = \infty$, if there exist constants $a_n > 0, b_n \in R$ and some non-degenerate distribution function G such that $(X_{n.k_n} - b_n)/a_n$ converges in distribution to G, then with probability one

$$\lim_{N\to\infty} \frac{1}{\log N} \sum_{n=1}^{N} \frac{1}{n} I\Big(\frac{X_{n,k_n} - b_n}{a_n} \leq x\Big) = G(x) \quad \text{for all } x \in C(G),$$

where $C(G)$ is the set of continuity points of G.

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