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ON THE EXISTENCE OF MOMENTS OF STOPPED SUMS IN MARKOV RENEWAL THEORY

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Abstract: Let $(M_n)_{n\geq 0}$ be an ergodic Markov chain on a general state space X with stationary distribution π and $g\colon X\to[0,\infty)$ a measurable function. Define $S_0(g)=0$ and $S_(g)\stackrel{def}{=}g(M_1)+\ldots+g(M_n)$ for $n\geq 1$. Given any stopping time T for $(M_n)_{n\geq 0}$ and any initial distribution ν for $(M_n)_{n\geq 0}$, the purpose of this paper is to provide suitable conditions for the finiteness of $E_\nu S_T(g)^p$ for $p>1$. A typical result states that

$$E_\nu S_T(g)^p\leq C_1(E_\nu S_T(g^p)+E_\nu T^p)+C_2$$

for suitable finite constants C_1, C_2. Our analysis is based to a large extent on martingale decompositions for $S_n(g)$ and on drift conditions for the function g and the transition kernel P of the chain. Some of the results are stated under the stronger assumption that $(M_n)_{n\geq 0}$ is positive Harris recurrent in which case stopping times T which are regeneration epochs for the chain are of particular interest. The important special case where $T=T(t)\stackrel{def}{=}\inf\{n\geq 1\colon S_n(g)>t\}$ for $t\geq 0$ is also treated.

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