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ON THE APPROXIMATION OF A RANDOM VARIABLE BY A CONDITIONAL EXPECTATION OF ANOTHER RANDOM VARIABLE

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Abstract: Let X and Y be $\mathbb{R}$-valued random variables on a non-atomic probability space $(\Omega, \mathcal{F}, P)$. We give conditions under which Y can be approximated by a conditional expectation of X. In particular, we prove the following theorem:

Let X be an $\mathbb{R}$-valued random variable such that $EX^+ = EX^- = \infty$. Then for each random variable Y and arbitrary $\epsilon > 0$ there exist $B \in \mathcal{F}$ and a sub-σ-field $\mathcal{S}$ of $\mathcal{F}$ such that $P(B) \leq \epsilon$ and $E(X|\mathcal{S}) = Y$ a.s. on B^c.

We also review some facts on the conditional expectation of unintegrable random variables.

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