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CURSE OF DIMENSIONALITY IN APPROXIMATION OF RANDOM FIELDS

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Abstract: Consider a random field of tensor product-type $X(t)$, $t \in [0,1]^d$, given by

$$X(t) = \sum_{k \in \mathbb{N}^d} \prod_{l=1}^{d} \lambda(k_l) \xi_k \prod_{l=1}^{d} \phi_{k_l}(t_l)$$

where $\big(\lambda(k_l)\big)_{i>0} \in l_2$, $(\phi_i)_{i>0}$ is an orthonormal system in $L_2[0,1]$ and $(\xi_k)_{k \in \mathbb{N}^d}$ are non-correlated random variables with zero mean and unit variance. We investigate the quality of approximation (both in the average and in the probabilistic sense) to X by the n-term partial sums X_n minimizing the quadratic error $E\|X - X_n\|^2$. In the first part of the paper we consider the case of fixed dimension d. In the second part, following the suggestion of H. Woźniakowski, we consider the same problem for $d \to \infty$. We show that, for any fixed level of relative error, approximation complexity increases exponentially and we find the explosion coefficient. We also show that the behavior of the probabilistic and average complexity is essentially the same in the large domain of parameters.

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