

**SHARP INEQUALITIES FOR THE HAAR SYSTEM AND MARTINGALE  
TRANSFORMS**

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*Abstract:* A classical result of Paley and Marcinkiewicz asserts that the Haar system on  $[0, 1]$  forms an unconditional basis in  $L^p$  provided  $1 < p < \infty$ . The purpose of the paper is to study related weak-type inequalities, which can be regarded as a version of this property for  $p = 1$ . Probabilistic counterparts, leading to some sharp estimates for martingale transforms, are presented.

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