PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 4, Fasc. 2 (1984), pp. 221–236

TWO APPROACHES TO CONSTRUCTING SIMULTANEOUS CONFIDENCE BOUNDS FOR QUANTILES

M. Csörgő
P. Révész

Abstract: Given some regularity conditions on the distribution function F of a random sample $X_1, X_2, ..., X_n$, the sequence of quantile processes $\{n^{1/2} f(Q(y))(Q_n(y) - Q(y)); 0 < y < 1\}$ behaves like a sequence of Brownian bridges $\{B_n(y); 0 < y < 1\}$, where $Q(y) := F^{-1}(y)$, the inverse of $F(\cdot)$, and $Q_n(y) = X_{k:n}$ if $(k-l)/n < y \leq k/n$ $(k = 1, 2, .., n)$ with the order statistics $X_{1:n} \leq X_{2:n} \leq ... \leq X_{n:n}$ of the above sample. First, a sequence of consistent direct estimators is proposed for the quantile-density function $Q'(y) = 1/f(Q(y))$. The latter then also enables us to construct simultaneous confidence bounds for an unknown quantile function $Q(y)$. The second approach makes frequently misused heuristic steps like

$$\begin{aligned} 1 - \alpha &= P\{F(x) - n^{-1/2}c(\alpha) \leq F_n(x) \leq F(x) + n^{1/2}c(\alpha); -\infty < x < \infty\} \\ &= P\{y - n^{-1/2}c(\alpha) \leq F_n(F^{-1}(y)) \leq y + n^{-1/2}c(\alpha); \\ &\qquad\qquad F^{-1}(0) < F^{-1}(y) < F^{-1}(1)\} \\ &= P\{F_n^{-1}(y - n^{-1/2}c(\alpha)) \leq F^{-1}(y) \leq F_n^{-1}(y + n^{-1/2}c(\alpha)); 0 < y < 1\} \end{aligned}$$

precise for large n, where F_n is the empirical distribution function of the above random sample, and for $\alpha \in (0,1),\ c(\alpha)$ is defined by

$$P\{\sup_{0 \leq y \leq 1} |B(y)| \leq c(\alpha)\} = 1 - \alpha$$

for a Brownian bridge $B(\cdot)$.

2000 AMS Mathematics Subject Classification: Primary: -; Secondary: -;

Key words and phrases: -

THE FULL TEXT IS AVAILABLE HERE