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ON CONVERGENCE OF L_1-BOUNDED MARTINGALES INDEXED BY DIRECTED SETS

Annie Millet
Louis Sucheston

Abstract: Let $(\mathcal{F}_t)$ be an increasing family of σ-algebras indexed by a directed set J. In this paper it is shown that every L_1-bounded real-valued martingale converges essentially if and only if a weak type of maximal inequality holds for all martingales. A new covering condition C stated in terms of multivalued stopping times is introduced and characterized in terms of maximal inequalities. C is shown to be strictly weaker than the Vitali condition V, than SV (see [15]), and also sigma-SV. Under C, L_1-bounded martingales taking values in a Banach space with the Radon-Nikodým property converge essentially.

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