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ON CANONICAL REPRESENTATIONS OF STABLE M_t-PROCESSES

BY

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Dedicated to Professor N. Tatsuuma
on the occasion of his sixtieth birthday

Abstract. Using integral geometry a symmetric α stable system is constructed as a stable analogue of Lévy's multiparameter Brownian motion. M_t-process is the spherical mean process of this stable system. In the case of odd dimension, a concrete example of canonical representation of M_t-process is obtained through this construction.

Hida–Cramér theory of canonical representations provides us useful tools to analyze Gaussian (or 2nd order) processes. It seems that their Gaussian theory highly depends on the tools from the theory of Hilbert spaces. Is it impossible to extend their theory of canonical representations to the processes without 2nd moment?

In the Gaussian case, the theory of canonical representations started from Lévy's investigations of representations of Gaussian M_t-processes. In this paper, using integral geometry we construct a stable analogue of multiparameter Lévy's Brownian motion. Next we define a stable version of M_t-process and obtain its canonical representation. This representation comes from this integral geometric construction in a natural manner. We hope our example will play a similar role in the representation theory of stable processes as Lévy's M_t-process plays in Hida–Cramér theory.

1. MULTIPARAMETER SαS LÉVY MOTION

In the Gaussian case the M_t-process is defined as spherical means of the multiparameter Brownian motion. Let us start with the definition of multiparameter Lévy motion which is a stable analogue of multiparameter Brownian motion.

Definition 1. A set of random variables $\{X(\lambda); \lambda \in \Lambda\}$ is called an $S\alpha S$-*system* (*Symmetric* α *Stable system*), $0<\alpha\leqslant 2$, if any finite linear combination $L=\sum a_i X(\lambda_i)$ is subject to the symmetric α stable law, that is, $\mathrm{E}[e^{izL}]=e^{-c|z|^\alpha}$ with a non-negative constant $c=c(a_i, \lambda_i)$, $z\in \boldsymbol{R}$.

Definition 2. An $S\alpha S$-system $\mathscr{Y}^\alpha=\{Y^\alpha(B); B\in\mathscr{B}, \mu(B)<\infty\}$, for $0<\alpha\leqslant 2$, is called an $S\alpha S$ *random measure* controlled by a measure space $(E, \mathscr{B}, \mu)$ if

(1) $\mathrm{E}[\exp\{izY^\alpha(B)\}]=\exp\{-\mu(B)|z|^\alpha\}$,

(2) the random variables $Y^\alpha(B_1), \dots, Y^\alpha(B_N)$ are mutually independent for any disjoint family $\{B_i\colon i=1,\dots,N\}$,

(3) $Y^\alpha(\bigcup_1^\infty B_i)=\sum_1^\infty Y^\alpha(B_i)$ a.e. for any disjoint family $\{B_i\in\mathscr{B}\}$ which satisfies $\mu(\bigcup B_i)<\infty$.

Let

$$\mathscr{H}_n=\{\text{a hyperplane of co-dimension 1 in } \boldsymbol{R}^n\}\simeq (S^{n-1}\times \boldsymbol{R}^1)/\sim, \tag{1.1}$$

where $\sim$ means the projective equivalence relation. The group of Euclidean solid motions $M(n)=\mathrm{SO}(n)\times\boldsymbol{R}^n$ acts on $\mathscr{H}_n$ in the natural manner and the measure $d\mu=d\boldsymbol{q}dp$ ($\boldsymbol{q}\in S^{n-1}$, $p\geqslant 0$) on the topological σ-field $\mathscr{B}=\mathscr{B}(\mathscr{H}_n)$ is invariant under the action of $M(n)$, where $d\boldsymbol{q}$ is the normalized uniform measure on S^{n-1}.

Set

$$S_{\boldsymbol{t}}=\{h\in\mathscr{H}_n;\ h \text{ separates the origin } \boldsymbol{O} \text{ and } \boldsymbol{t}\},\qquad \boldsymbol{t}\in\boldsymbol{R}^n. \tag{1.2}$$

Theorem 1 (Chentsov [2]). *Let $\mathscr{Y}_n^2$ be the Gaussian random measure with control measure space $(\mathscr{H}_n, \mathscr{B}, \mu)$. Then the Gaussian system $\{B(\boldsymbol{t}); \boldsymbol{t}\in\boldsymbol{R}^n\}$ defined by*

$$B(\boldsymbol{t})=Y_n^2(S_{\boldsymbol{t}}) \tag{1.3}$$

satisfies

(1) $B(\boldsymbol{O})=0$, *and*

(2) $\mathrm{E}[\exp\{i(B(\boldsymbol{t})-B(\boldsymbol{s}))z\}]=\exp\{-c\|\boldsymbol{t}-\boldsymbol{s}\|\cdot|z|^2\}$, *where* $\|\cdot\|$ *means the Euclidean norm and*

$$c=\begin{cases}\frac{1}{2}, & n=1,\\ \{2(n-1)\int_0^{\pi/2}\sin^{n-2}\theta d\theta\}^{-1}, & n\geqslant 2.\end{cases}$$

P. Lévy calls the above system $\{B(\boldsymbol{t});\ \boldsymbol{t}\in\boldsymbol{R}^n\}$ the *$\boldsymbol{R}^n$-parameter Brownian motion with respect to the metric function* $2c\|\cdot\|$. It is natural to define the following $S\alpha S$-analogue:

$$X_n^\alpha(\boldsymbol{t})=Y_n^\alpha(S_{\boldsymbol{t}}),\qquad 0<\alpha\leqslant 2,\ \boldsymbol{t}\in\boldsymbol{R}^n. \tag{1.4}$$

Here $\mathscr{Y}_n^\alpha=\{Y_n^\alpha\}$ is the $S\alpha S$ random measure with control measure space $(\mathscr{H}_n, \mathscr{B}, \mu)$.

Then

PROPOSITION 1. *The above family of random variables* $\{X_n^\alpha(t)\}$ *becomes an* SαS-*system and satisfies*

(1) $X_n^\alpha(O)=0$;

(2) $\{X_n^{\alpha,g}(t)\equiv X_n^\alpha(gt)-X_n^\alpha(gO)\}$ *and* $\{X_n^\alpha(t)\}$ *share the same finite dimensional laws for any* $g\in M(n)$;

(3) *for any* e_0 *and* $e_1\in R^n$, *the restriction of the process* $\{X_n^\alpha(t)\}$ *on the line* $l=\{e_0+te_1;\ t\in R^1\}$

$$X^l(t)\equiv X_n^\alpha(e_0+te_1)-X_n^\alpha(e_0),\qquad t\in R^1,$$

is a 1-*parameter* SαS-*process with stationary independent increments, that is, an* SαS *Lévy motion or, equivalently, a* 1-*parameter Lévy process of index* α *in the restricted sense.*

Note that from (2) and (3) we have

$$\mathrm{E}\left[\exp\{i(X_n^\alpha(t)-X_n^\alpha(s))z\}\right]=\exp\{-c\,\|t-s\|\cdot|z|^\alpha\}. \tag{1.5}$$

In the case of $0<\alpha<2$, Mori [11] proved the uniqueness of the process which satisfies (1)–(3), so it is natural to call this SαS-system $\{X_n^\alpha(t);\ t\in R^n\}$ the R^n-*parameter* SαS *Lévy motion.*

For convenience, let us rewrite the definition (1.4) in the following integral form:

$$X_n^\alpha(t)=\int\limits_{0<p\leqslant\langle t,q\rangle} Y_n^\alpha(dq\,dp), \tag{1.6}$$

where $\langle\cdot,\cdot\rangle$ means the inner product in R^n.

2. $M_n^\alpha(t)$-PROCESS AND ITS CANONICAL REPRESENTATION

Let us recall the Gaussian case. P. Lévy introduced a Gaussian process $M_n(t)$, $t>0$, as the spherical mean of multiparameter Brownian motion $B(t)$, $t\in R^n$,

$$M_n(t)=\int\limits_{\|t\|=t} B(t)\,d(t/t). \tag{2.1}$$

And he noticed that this Gaussian process can be represented in several non-equivalent forms as

$$M_n(t)=\int_0^t F(t,u)\,dB(u), \tag{2.2}$$

where $B(u)$ is a standard (1-parameter) Brownian motion (see [4], [8], [9]).

The theory of canonical representations clarifies the above fact.

DEFINITION 3. The representation of Gaussian process

$$A(t) = \int_0^t F(t, u)dB(u) \tag{2.3}$$

is called *proper canonical* if

$$\mathscr{M}(A(s); s \leqslant t) = \mathscr{M}(B(s); s \leqslant t) \quad \text{for any } t > 0, \tag{2.4}$$

where $\mathscr{M}$ means the closed linear hull in L^2-sense.

THEOREM 2 (Karhunen [6]). *There exists a unique proper canonical representation for any separable, mean continuous and purely non-deterministic stationary Gaussian processes.*

We can apply this theorem to the process $M_n(t)$ by time change and normalization. We see that one of Lévy's examples of the representations of the form (2.2) is proper canonical.

In this section we consider an SαS-analogue of $M_n(t)$-process and obtain its proper canonical representation.

2.1. SαS $M_n^\alpha(t)$-processes. Let us introduce the following metric space:

$$L^{(\alpha)} = \{f\text{: a measurable function on } (E, \mu);\ \int_E |f|^\alpha d\mu < \infty\} \tag{2.5}$$

with metric $(\int_E |f|^\alpha d\mu)^{(1/\alpha)\wedge 1}$. Then the S$\alpha$S Wiener integral $I^\alpha(f)$ of the element f of $L^{(\alpha)}$ is defined as the limit in probability of the sequence of SαS random variables $\{\sum_i a_i^n Y^\alpha(B_i^n), n = 1, 2, \ldots\}$, where $\{f_n = \sum a_i^n \chi_{B_i^n}\}$ is a sequence of simple functions which converges to f in $L^{(\alpha)}$. Using this relation let us induce a metric from $L^{(\alpha)}$ in the space of SαS random variables of the form $I^\alpha(f), f \in L^{(\alpha)}$, and identify these two spaces (see [5], [15], [19]).

Consider the spherical mean of the multiparameter Lévy motion:

$$M_n^\alpha(t) = \int_{\|\boldsymbol{t}\| = t} X_n^\alpha(\boldsymbol{t}) d(\boldsymbol{t}/t), \tag{2.6}$$

where the right-hand side means the limit of Riemannian sum in $L^{(\alpha)}$

$$\sum_1^N a_i^N \int_{\mathscr{H}_n} \chi_{\{\langle \boldsymbol{t}_i, \boldsymbol{q}\rangle \geqslant p\}}(\boldsymbol{q}, p)\, Y_n^\alpha(d\boldsymbol{q}dp), \qquad n \geqslant 2. \tag{2.7}$$

In the case of $n = 1$,

$$M_1^\alpha(t) = \tfrac{1}{2}\big(X_1^\alpha(t) + X_1^\alpha(-t)\big).$$

For $n \geqslant 2$, set

$$F_N(t; \boldsymbol{q}, p) = \sum_1^N a_i^N \chi_{\{\langle \boldsymbol{t}_i, \boldsymbol{q}\rangle \geqslant p\}}(\boldsymbol{q}, p); \tag{2.8}$$

then

$$(2.9)\qquad F_N(t;\, \boldsymbol{q},\, p) \to c_0^n \cdot F(t,\, p) = c_0^n \int_{p/t}^{1} (1-x^2)^{(n-3)/2}\, dx \quad \text{in } L^{(\alpha)},$$

where

$$c_0^n = \Big(\int_0^{\pi} \sin^{n-2}\theta d\theta\Big)^{-1} = \frac{\Gamma(n/2)}{\sqrt{\pi}\,\Gamma((n-1)/2)}.$$

For $n = 1$ we have $F_N \equiv F(t,\, p) = \frac{1}{2}$.

Now we can rewrite the definition of M_n^α as

$$(2.10)\qquad M_n^\alpha(t) = c_0^n \int_{S^{n-1}\times[0,t]} F(t,\, p)\, Y_n^\alpha(d\boldsymbol{q}dp) = c_0^n \int_0^t F(t,\, p) dZ^\alpha(p),$$

where

$$(2.11)\qquad Z^\alpha(p) = Y_n^\alpha(S^{n-1}\times[0,\, p]).$$

The SαS-process Z^α has stationary independent increments, that is, Z^α is an SαS Lévy motion.

2.2. Main result.

THEOREM 3 (for the Gaussian case, see [10] and [18]). *If n is odd and $0 < \alpha \leqslant 2$, then the representation*

$$(2.12)\qquad M_n^\alpha(t) = c_0^n \int_0^t F(t,\, p) dZ^\alpha(p),$$

where

$$(2.13)\qquad F(t,\, p) = \int_{p/t}^{1} (1-x^2)^{(n-3)/2}\, dx,$$

satisfies

$$(2.14)\qquad \mathcal{M}^\alpha\big(M_n^\alpha(s);\, s \leqslant t\big) = \mathcal{M}^\alpha\big(Z^\alpha(s);\, s \leqslant t\big) \quad \textit{for any } t > 0,$$

where $\mathcal{M}^\alpha$ means the closed linear hull in $L^{(\alpha)}$.

The representation (2.12) which satisfies (2.14) is unique in Hida's sense (for the Gaussian case see [4], in general [7]). Because of the above equality, we may call a representation of the form (2.12) which satisfies the relation (2.14) *proper canonical.* For even n, see Section 3.2 of this paper.

Proof. If $n = 1$, the theorem is trivial. In the case of $n = 3$, consider

$$tM_3^\alpha(t) = \tfrac{1}{2}\int_0^t (t-p) dZ^\alpha(p)$$

and take the difference

$$(t+h)M_3^\alpha(t+h)-tM_3^\alpha(t)=\tfrac{1}{2}\Big(\int_0^t h\,dZ^\alpha(p)+\int_t^{t+h}(t+h-p)\,dZ^\alpha(p)\Big).$$

Then we have

$$h^{-1}\big((t+h)M_3^\alpha(t+h)-tM_3^\alpha(t)\big)=\tfrac{1}{2}\Big(\int_0^t dZ^\alpha(p)+\int_t^{t+h}O(1)\,dZ^\alpha(p)\Big).$$

Thus, the integrand converges in $L^{(\alpha)}$ as $h\to 0$. So $tM_3^\alpha(t)$ is differentiable in $L^{(\alpha)}$ and we have

$$\frac{d}{dt}tM_3^\alpha(t)=\frac{1}{2}Z^\alpha(t) \quad \text{in } L^{(\alpha)}.$$

For $n\geqslant 5$, we can reduce the proof to the case of $n=3$ by the next lemma.

LEMMA. *Set*

$$D_n=t^{-n+3}\,\frac{d}{dt}t^{n-2};$$

then we can apply this operator to $M_n^\alpha(t)$ in $L^{(\alpha)}$ and we obtain

$$D_nM_n^\alpha(t)=(n-2)M_{n-2}^\alpha(t) \text{ in law}$$

as stochastic processes.

Proof. Set $N(t)\equiv t^{n-2}M_n^\alpha(t)$. Then

$$N(t)=c_0^n\int_0^t t^{n-2}\int_{p/t}^1(1-x^2)^{(n-3)/2}dx\,dZ^\alpha(p)=c_0^n\int_0^t\int_p^t(t^2-x^2)^{(n-3)/2}dx\,dZ^\alpha(p).$$

The difference is

$$\begin{aligned}&h^{-1}\big(N(t+h)-N(t)\big)\\&=c_0^n\int_0^{t+h}h^{-1}\Big\{\int_p^{t+h}\big((t+h)^2-x^2\big)^{(n-3)/2}dx-\chi_{[0,t]}(p)\int_p^t(t^2-x^2)^{(n-3)/2}dx\Big\}dZ^\alpha(p).\end{aligned}$$

The $L^{(\alpha)}$-metric of the above integral is

$$\begin{aligned}&\Big[\int_0^{t+h}h^{-\alpha}\Big|\int_p^{t+h}\big((t+h)^2-x^2\big)^{(n-3)/2}dx-\chi_{[0,t]}(p)\int_p^t(t^2-x^2)^{(n-3)/2}dx\Big|^\alpha dp\Big]^{(1/\alpha)\wedge 1}\\
&\leqslant\Big[h^{-\alpha}\int_t^{t+h}\Big|\int_p^{t+h}\big((t+h)^2-x^2\big)^{(n-3)/2}dx\Big|^\alpha dp\Big]^{(1/\alpha)\wedge 1}\\
&\quad+\Big[h^{-\alpha}\int_0^t\Big|\int_p^t\Big(\big((t+h)^2-x^2\big)^{(n-3)/2}-(t^2-x^2)^{(n-3)/2}dx\\
&\quad+\int_t^{t+h}\big((t+h)^2-x^2\big)^{(n-3)/2}dx\Big|^\alpha dp\Big]^{(1/\alpha)\wedge 1}\\
&=\big[O(h^{\alpha(n-3)/2+1})\big]^{(1/\alpha)\wedge 1}+\big[O(1)\big]^{(1/\alpha)\wedge 1}=O(1).\end{aligned}$$

The process $N(t)$ is differentiable in $L^{(\alpha)}$, and we have

$$\frac{d}{dt} t^{n-2} M_n^\alpha(t) = c_0^n (n-3) t \int_0^t \int_p^t (t^2 - x^2)^{(n-5)/2} dx\, dZ^\alpha(p)$$

$$= (n-3)\frac{c_0^n}{c_0^{n-2}} t^{n-3} M_{n-2}^\alpha(t) \text{ in law.}$$

The constant $(n-3)(c_0^n/c_0^{n-2})$ is equal to

$$(n-3)\frac{\Gamma(n/2)}{\Gamma((n-1)/2)}\frac{\Gamma((n-3)/2)}{\Gamma((n-2)/2)} = n-2.$$

Thus

$$t^{-n+3}\frac{d}{dt}\left(t^{n-2} M_n^\alpha(t)\right) = (n-2) M_{n-2}^\alpha(t). \quad \blacksquare$$

3. GENERALIZATION OF PARAMETER SPACE AND REMARKS

In the Gaussian case it seems that McKean already knew the idea we employ in this paper – the relation between Chentsov's construction of multiparameter Brownian motion and the canonical representation of M_t-process (see [10]). In this paper we clarified this idea and show that this relation holds also in the stable case. In this section we show one direct generalization of the results of Section 2 and two further investigations.

3.1. Generalization of parameter space. The results of this paper are easy to extend to the spaces of constant curvature (for the Gaussian case see [18]). We present here the required definitions and our results. We omit the proofs to avoid unnecessary duplications, they can be obtained in almost the same manner as in Section 2.

Let R_κ be S^n, $\boldsymbol{R}^n$ or the n-dimensional hyperbolic space H^n, respectively, as $\kappa = 1, 0, -1$ and let

(3.1) $\quad \mathscr{H}_\kappa^n = \{$a totally geodesic submanifold of R_κ of co-dimension 1$\}$.

The group G_κ, $SO(n+1)$, $M(n)$ or the hyperbolic group L_n, respectively, acts on the space R_κ. Let $\mathscr{Y}_{n,\kappa}^\alpha$ be the SαS random measure controlled by the invariant measure μ_κ^α on $\mathscr{H}_\kappa^n$.

Set

(3.2) $\quad S_t = \{$an element of $\mathscr{H}_\kappa^n$ which separates the origin O and $t\}$.

Then the SαS-system

(3.3) $$X_\kappa^\alpha(t) \equiv Y_{n,\kappa}^\alpha(S_t)$$

satisfies

(1) $X_\kappa^\alpha(O) = 0$,

(2) $\{X_\kappa^{\alpha,g}(t) \equiv X_\kappa^\alpha(gt) - X_\kappa^\alpha(gO)\}$ and $\{X_\kappa^\alpha(t)\}$ share the same finite dimensional laws for any $g \in G_\kappa$,

(3) $\mathrm{E}[\exp(izX_\kappa^\alpha(t))] = \exp(-c_\kappa d_\kappa(t, O)|z|^\alpha)$, c_κ is a positive constant and d_κ is the geodesic metric of R_κ.

Thus we can call the above process the *R_κ-parameter Lévy motion.*

Take the spherical mean of X_κ^α; then we obtain an analogue of $M^\alpha(t)$-process:

$$M_\kappa^\alpha(t) \equiv \int_{d_\kappa(O,t)=t} X_\kappa^\alpha(t) d(t/t), \tag{3.4}$$

where d_κ is the G_κ-invariant geodesic metric in R_κ. Using the same idea as we use in Section 2 we have

THEOREM 4. *If n is odd, then the representation of the* SαS*-process $M_\kappa^\alpha(t)$,*

$$M_\kappa^\alpha(t) = c_\kappa^n \int_0^t F(t, p) dL_\kappa^\alpha(p), \quad c_\kappa^n \text{ is a constant,} \tag{3.5}$$

is proper canonical in the meaning of Theorem 3. *Here the representation kernel $F(t, p)$ is the same one as* (2.13) *and the process $L_\kappa^\alpha(p)$ is an independent increment* SαS*-process which satisfies*

$$\mathrm{E}[\exp(iL_\kappa^\alpha(p)z)] = \begin{cases} \exp(-\int_0^{\tan^{-1}(p)} \cos^{n-1}(\theta) d\theta \cdot |z|^\alpha), & \kappa = 1, \\ \exp(-p \cdot |z|^\alpha), & \kappa = 0, \\ \exp(-\int_0^{\tanh^{-1}(p)} \cosh^{n-1}(u) du \cdot |z|^\alpha), & \kappa = -1. \end{cases} \tag{3.6}$$

3.2. Further results on even n and on weighted means. The first-named author obtained the following results in his master thesis (Kojo [7]):

(i) The case of even n, where the representations (2.10) and (3.5) are also proper canonical, includes the Gaussian case (for the Gaussian case, see [10]).

(ii) Let us consider the generalized M_t-processes which are defined as weighted means of SαS-motion on spheres with higher order spherical harmonics as the weight. We can apply the idea of Section 2 to obtain their representations. If $\alpha \neq 2$, the non-Gaussian case, some of representations of generalized M_t-processes are not proper canonical but non-linearly canonical.

Here we have to distinguish two canonicalities. One is proper canonical which we introduce in Section 2, the other is non-linearly canonical, that is, two σ-fields (of the process in question and of the process with independent increments which appears in the integral) are equal to each other. In the

Gaussian case two notions of canonicality are equivalent. The result (ii) means that in the SαS case there exist concrete examples of representations which are not proper canonical but non-linearly canonical.

3.3. Remarkable properties of a class of SαS-processes. As we saw the multiparameter SαS-motion is constructed by an integral geometric idea. In general, non-Gaussian SαS random fields defined by integral geometry have remarkable properties:

(i) Their finite dimensional distributions have a pure point spectrum for any dimension (see [12] and [13]).

(ii) They follow strong determinisms. For instance, we have a class of n-parameter SαS random fields which are determined by their $(n+1)$-dimensional marginals. Especially, a 1-parameter SαS-process which belongs to such a class is, like a Gaussian process, determined by its 2-dimensional marginal distributions. We have also an example of a 1-parameter SαS-process which is determined by its 4-dimensional marginals but not 3-dimensional marginals (see [13] and [14]).

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