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RADONIFYING OPERATORS RELATED TO p-STABLE MEASURES ON BANACH SPACES

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Abstract. For a real p $(1 < p < 2)$ and its conjugate p' we characterize Banach spaces E for which an operator $T\colon L_{p'} \to E$ is θ_p-Radonifying iff T' is p-absolutely summing. In case $p = 2$ these are exactly spaces of type 2 as was proved by Chobanjan and Tarieladze [1]. For $p < 2$ the condition is much stronger because these are spaces of stable type p isomorphic to a subspace of some L_p.

Let E be a real Banach space. For a real number p $(1 < p < 2)$ let L_p be a separable Banach space of measurable functions having p-integrable absolute value. Let $1/p + 1/p' = 1$. An operator T from $L_{p'}$ into E is said to be *θ_p-Radonifying* if $\exp(-\|T'a\|^p)$ is the characteristic function of a Radon measure μ on E. Here θ_p is a cylindrical measure on $L_{p'}$ with the characteristic function of the form $\exp(-\|g\|^p)$, $g \in L_p$. Thus T is θ_p-Radonifying iff $T(\theta_p)$ extends to a Radon measure on E. In this case the Radon extension is a p-stable symmetric measure on E. It turns out that the set $\Sigma_p(L_{p'}, E)$ of all θ_p-Radonifying operators becomes a Banach space under the equivalent norms

$$\sigma_{pr}(T) = \Big(\int_E \|x\|^r \, d\mu\Big)^{1/r}, \quad 1 \leqslant r < p < 2.$$

Let us recall that E is of *stable type* p if there exists a constant $c > 0$ such that for all $x_1, x_2, \ldots, x_n \in E$

$$\Big(\mathrm{E}\,\Big\|\sum_{i=1}^{n} x_i \xi_i\Big\|^r\Big)^{1/r} \leqslant c\Big(\sum_{i=1}^{n} \|x_i\|^p\Big)^{1/p}$$

for each r with $0 < r < p$, where $\xi_1, \xi_2, \ldots, \xi_n$ is a sequence of i.i.d. random variables with characteristic function $\exp(-|t|^p)$.

If E and F are Banach spaces, then the operator $T\colon E \to F$ is called *p-absolutely summing* ($T \in \Pi_p(E, F)$) if for some constant M and for each $x_1, x_2, \ldots, x_n \in E$ the inequality

$$\sum_{i=1}^{n} \|Tx_i\|^p \leqslant M^p \sup_{x' \in E', \|x'\| \leqslant 1} \sum_{i=1}^{n} |\langle x_i, x' \rangle|^p$$

holds. Denote by $\pi_p(T)$ the least such constant M.

The following relation between the θ_p-Radonifying operators and the p-absolutely summing operators is known in a more general version as the celebrated L. Schwartz's duality theorem (cf. [2] and references therein):

PROPOSITION. *If* $T \in \Sigma_p(L_{p'}, E)$, *then* $T' \in \Pi_p(E', L_p)$.

The converse implication does not hold in general. In the following theorem we characterize Banach spaces for which it holds.

THEOREM. *Let* $1 < p < 2$. *Then the following two conditions on a Banach space* E *are equivalent*:

(1) $T \in \Sigma_p(L_{p'}, E)$ *if* $T' \in \Pi_p(E', L_p)$ *for each space* L_p.

(2) E *is of stable type* p *and isomorphic to a subspace of some* L_p.

Proof. (1) $\Rightarrow$ (2). Let $x_1, x_2, \ldots, x_n \in E$. We define an operator T from $L_{p'}$ into E by

$$\|T' a\|^p = \sum_{i=1}^{n} |\langle x_i, a \rangle|^p.$$

Condition (1) implies the existence of a constant $c > 0$ such that $\sigma_{pr}(T) \leqslant c\pi_p(T')$. In addition, let us observe that the characteristic function of the p-stable measure μ defined by T is equal to the characteristic function of the distribution of the E-valued random vector $\sum_{i=1}^{n} x_i \xi_i$. Namely,

$$\hat{\mu}(a) = \exp(-\|T' a\|^p) = \exp\Big(-\sum_{i=1}^{n} |\langle x_i, a \rangle|^p\Big).$$

Thus we have

$$\Big(\mathrm{E}\,\Big\|\sum_{i=1}^{n} x_i \xi_i\Big\|^r\Big)^{1/r} = \Big(\int_E \|x\|^r\, d\mu\Big)^{1/r} = \sigma_{pr}(T) \leqslant c\pi_p(T') \leqslant c\Big(\sum_{i=1}^{n} \|x_i\|^p\Big)^{1/p},$$

which shows that E is of stable type p.

To prove that the space E is isomorphic to a subspace of some L_p we choose $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ belonging to E with the property

$$\sum_{i=1}^{n} |\langle x_i, a \rangle|^p \leqslant \sum_{i=1}^{n} |\langle y_i, a \rangle|^p \quad \text{for all } a \in E'.$$

Now we define operators T and S from $L_{p'}$ into E by

$$\|T'a\|^p = \sum_{i=1}^{n} |\langle x_i, a\rangle|^p \quad \text{and} \quad \|S'a\|^p = \sum_{i=1}^{n} |\langle y_i, a\rangle|^p \quad \text{for all } a \in E'.$$

The inequality $\|T'a\| \leqslant \|S'a\|$ for all $a \in E'$ implies $\pi_p(T') \leqslant \pi_p(S')$. Since each Banach space is of stable cotype p for $p < 2$, we have

$$\Big(\sum_{i=1}^{n} \|x_i\|^p\Big)^{1/p} \leqslant c_1 \Big(\mathrm{E}\,\Big\|\sum_{i=1}^{n} x_i \xi_i\Big\|^r\Big)^{1/r} = c_1 \sigma_{pr}(T)$$

$$\leqslant c_2 \pi_p(T') \leqslant c_2 \pi_p(S') \leqslant c_2 \Big(\sum_{i=1}^{n} \|y_i\|^p\Big)^{1/p}.$$

By Lindenstrauss-Pełczyński's theorem ([4], Theorem 7.3) we claim that E is isomorphic to a subspace of some L_p.

(2) ⇒ (1). Consider an operator $T\colon L_{p'} \to E$ such that T' is p-absolutely summing. Since E is isomorphic to a subspace of some L_p, by Kwapień's theorem [3] we have $T \in \Pi_p(L_{p'}, E)$. It follows from separability of the space L_p that there exists an isometric imbedding J from L_p into $L_p[0, 1]$. Then TJ' is p-absolutely summing. By Kwapień's theorem [2] there exists a strongly measurable function φ from $[0, 1]$ into E with $\mathrm{E}\|\varphi\|^p < \infty$ such that

$$\|T'a\|^p = \|JT'a\|^p = \int_0^1 |\langle \varphi(t), a\rangle|^p\, dt.$$

Since E is of stable type p, $\exp(-\|T'a\|^p)$ is (by [5]) the characteristic function of a Radon measure, i.e., $T \in \Sigma_p(L_{p'}, E)$, which completes the proof.

COROLLARY. *Let $1 < p < 2$. Then the following two conditions on a Banach space E are equivalent:*

(1) *$T \in \Sigma_p(L_{p'}, E)$ if and only if $T' \in \Pi_p(E', L_p)$ for each space L_p.*

(2) *E is of stable type p and isomorphic to a subspace of some L_p.*

Remark. It is known by Rosenthal's theorem (see [7]) that condition (2) is equivalent to each of the following ones:

(3) *E is isomorphic to a subspace of some L_p and does not contain an isomorphic copy of l_p.*

(4) *E is isomorphic to a subspace of some L_p and there exists a real r $(0 < r < p)$ such that the topologies of L_p and L_r coincide on E.*

(5) *There exists a real q $(p < q \leqslant 2)$ such that E is isomorphic to a subspace of some L_q.*

Added in proof. After this note was completed, the authors were made aware of the paper of D. H. Thang and N. D. Tien, *On the extension of stable cylindrical measures*, Acta Math. Vietnam. 5 (1980), p. 169-177, where an equivalent result was established. Methods of proofs are however different. We also refer the reader to our paper *p-stable measures and p-absolutely summing operators*, p. 167-178 in: Lecture Notes in Math. 828 Springer Verlag, 1980, for some additional results on this subject.

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