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# DISCRETE PROBABILITY MEASURES ON $2 \times 2$ STOCHASTIC MATRICES AND A FUNCTIONAL EQUATION ON $[0,1]$ 

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Abstract: In this paper, we consider the following natural problem: suppose $\mu_{1}$ and $\mu_{2}$ are two probability measures with finite supports $S\left(\mu_{1}\right), S\left(\mu_{2}\right)$ respectively, such that $\left|S\left(\mu_{1}\right)\right|=\left|S\left(\mu_{2}\right)\right|$ and $S\left(\mu_{1}\right) \cup S\left(\mu_{2}\right) \subset 2 \times 2$ stochastic matrices, and $\mu_{1}^{n}$ (the $n$-th convolution power of $\mu_{1}$ under matrix multiplication), as well as $\mu_{2}^{n}$, converges weakly to the same probability measure $\lambda$, where $S(\lambda) \subset 2 \times 2$ stochastic matrices with rank one. Then when does it follow that $\mu_{1}=\mu_{2}$ ? What if $S\left(\mu_{1}\right)=S\left(\mu_{2}\right)$ ? In other words, can two different random walks, in this context, have the same invariant probability measure? Here, we consider related problems.

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