PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 24, Fasc. 2 (2004), pp. 213–235

ADAPTIVE KERNEL ESTIMATION OF THE MODE IN A NONPARAMETRIC RANDOM DESIGN REGRESSION MODEL

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Abstract: In a nonparametric regression model with random design, where the regression function m is given by $m(x) = E(Y|X = x)$, estimation of the location θ (*mode*) and size $m(\theta)$ of a unique maximum of m is considered. As estimators, location $\hat{\theta}$ and size $\hat{m}(\hat{\theta})$ of a maximum of the Nadaraya-Watson kernel estimator $\hat{m}$ for the curve m are chosen. Within this setting, we establish joint asymptotic normality and asymptotic independence for $\hat{\theta}$ and $\hat{m}(\hat{\theta})$ (which can be exploited for constructing simultaneous confidence intervals for θ and $m(\theta)$) under mild local smoothness assumptions on m and the design density g (imposed in a neighborhood of θ). The bandwidths employed for $\hat{m}$ are data-dependent and of plug-in type. This is handled by viewing the estimators as stochastic processes indexed by a so-called scaling parameter and proving functional central limit theorems for those processes. In the same way, we obtain, as a by-product, an asymptotic normality result for the Nadaraya-Watson estimator itself at a finite number of distinct points, which improves on previous results.

2000 AMS Mathematics Subject Classification: 62G05, 62G07.

Key words and phrases: Nonparametric regression, random design, mode, kernel smoothing, Nadaraya-Watson estimator, weak convergence, functional central limit theorems.