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CAUCHY TRANSFORMS OF MEASURES VIEWED AS SOME FUNCTIONALS OF FOURIER TRANSFORMS

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Abstract: The Cauchy transform of a positive measure plays an important role in complex analysis and more recently in so-called free probability. We show here that the Cauchy transform restricted to the imaginary axis can be viewed as the Fourier transform of some corresponding measures. Thus this allows the full use of that classical tool. Furthermore, we relate restricted Cauchy transforms to classical compound Poisson measures, exponential mixtures, geometric infinite divisibility and free-infinite divisibility. Finally, we illustrate our approach with some examples.

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