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SMALL DEVIATION OF SUBORDINATED PROCESSES OVER COMPACT SETS

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Abstract: Let $A = \big(A(t)\big)_{t\geq 0}$ be a subordinator. Given a compact set $K \subset [0,\infty)$ we prove two-sided estimates for the covering numbers of the random set $\{A(t) : t \in K\}$ which depend on the Laplace exponent Φ of A and on the covering numbers of K. This extends former results in the case $K = [0,1]$. Using this we find the behavior of the small deviation probabilities for subordinated processes $\Big(W_H\big(A(t)\big)\Big)_{t\in K}$, where W_H is a fractional Brownian motion with Hurst index $0 < H < 1$. The results are valid in the quenched as well as in the annealed case. In particular, those questions are investigated for Gamma processes. Here some surprising new phenomena appear. As application of the general results we find the behavior of $\log P(\sup_{t\in K} |Z_\alpha(t)| < \varepsilon)$ as $\varepsilon \to 0$ for the α-stable Lévy motion Z_α. For example, if K is a self-similar set with Hausdorff dimension $D > 0$, then this behavior is of order $-\varepsilon^{-\alpha D}$ in complete accordance with the Gaussian case $\alpha = 2$.

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